245: Fundamentals of Statistics

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Problem Set #2 Fall 2021

Due Wednesday, September 22, 2021 at 11:30pm on Canvas.

Please include your written answers, R code, and figures (when applicable). Please do not write your name in your homework submission. Upload your assignment as a single PDF file on Canvas with the format "PS2_XX.pdf", where XX is your assigned anonymous ID. Find your anonymous ID number in the 'Grades' tab on Canvas. Look at the 'Score' column of the 'Anonymous ID for Homeworks' row.

1. Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

   (a) What is the probability that a randomly selected adult regularly consumes both coffee and soda?

   (b) If a randomly selected adult regularly consumes soda, what is the probability that he/she also regularly consumes coffee?

   (c) What is the probability that a randomly selected adult does not regularly consume at least one of these two products?

2. A friend is giving a dinner party. His current wine supply includes 8 bottles of zinfandel, 10 of merlot and 12 of cabernet. Pick 6 bottles at random.

   (a) What is the probability of getting two bottles of each variety?

   (b) What is the probability of getting all of the same variety?

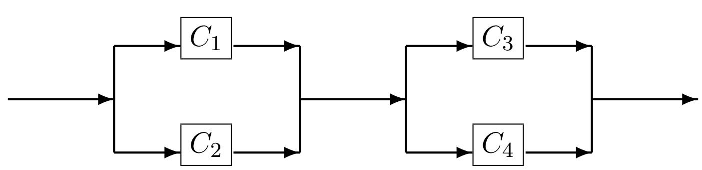
   (c) What is the probability of getting at least two cabernet?

   (d) What is the probability of getting the first merlot at the fourth pick?

3. Suppose that 30% of computer owners use a Macintosh, 50% use Windows, and 20% use Linux. Suppose that 65% of the Mac users have succumbed to a computer virus, 82% of the windows users get the virus, and 30% of the Linux users get the virus. We select a person at random.

   (a) What is the probability that her computer has infected by the virus?

   (b) Given the condition that her system has already infected by the virus, what is the probability that she is a Windows user?



4. Suppose that a system consists of 4 independent components $C_1, \cdots, C_4$ connected as below, with $P(C_1) = 0.9, P(C_2) = 0.8, P(C_3) = 0.95, P(C_4) = 0.7$, in which $P(C_i)$ denotes the probability that the component C_i works properly.

(a) What is the chance that the first parallelly connected component works properly?

(b) What is the probability that the system works properly?

5. Consider adjusted closing prices of SP500 index from Jan. 1, 2000 to September 8, 2016. Pick a day at random.

   (a) What is the probability that the SP500 is down? [Hint: Let `ndays = length(rSP500)` be the number of days and the probability is then `sum(rSP500 < 0)/ndays`].

   (b) What is the probability that the SP500 is down given its previous day is down? Are the signs of the returns of two consecutive days approximately independent?

   [Hint: `sum(rSP500[1:(ndays-1)]<0 & rSP500[2:(ndays)]<0)` is the number of two consecutive downs and `sum(rSP500[1:(ndays-1)]<0` is the number of previous days that the SP500 index is down.]

   (c) What is the probability that the absolute value of returns of the SP500 on the day is at least 1.5%?

   (d) If the absolute value of the return of SP500 is at least 1% in the previous close, what is the probability that the absolute value of the return of today is at least 1.5%?

   Remark: The first two problems are designed to show inpredictability of stock returns and the next two problems are intended to illustrate the dependence (predictability) of stock volatility.