Option Pricing with Model-guided Nonparametric Methods *

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Abstract

Parametric option pricing models are largely used in Finance. These models capture several features of asset price dynamics. However, their pricing performance can be significantly enhanced when they are combined with nonparametric learning approaches that learn and correct empirically the pricing errors. In this paper, we propose a new nonparametric method for pricing derivatives assets. Our method relies on the state price distribution instead of the state price density because the former is easier to estimate nonparametrically than the latter. A parametric model is used as an initial estimate of the state price distribution. Then the pricing errors induced by the parametric model are fitted nonparametrically. This model-guided method estimates the state price distribution nonparametrically and is called Automatic Correction of Errors (ACE). The method is easy to implement and can be combined with any model-based pricing formula to correct the systematic biases of pricing errors. We also develop a nonparametric test based on the generalized likelihood ratio to document the efficacy of the ACE method. Empirical studies based on S&P 500 index options show that our method outperforms several competing pricing models in terms of predictive and hedging abilities.

Keywords: Nonparametric regression, state price distribution, model misspecification, out-of-sample analysis, generalized likelihood ratio test.

JEL Classifications: C14, G13.

1 Introduction

Over the last three decades, there have been substantial efforts in extending the Black and Scholes (1973) model along several directions. These efforts aim at developing more flexible dynamics of asset prices leading to more accurate option pricing formulae. Examples include the jump-diffusion models of Bates (1991), and Madan, Carr, and Chang (1998); the stochastic volatility models of Hull and White (1987), and Heston (1993); the stochastic volatility and stochastic interest rates models of Amin and Ng (1993), Bakshi and Chen (1997); the stochastic volatility jump-diffusion models of Bates (1996), and Scott (1997), among others. These models have substantially relaxed the restrictions in the seminal work of Black and Scholes and made the assumptions of the price movements more plausible. However, these models are not derived from comprehensive economic theories, often rely on different assumptions concerning the risk neutral asset dynamics, and need to be simple and convenient to allow for the derivation of pricing formulae. Hence these models cannot be expected to capture all the relevant features of the involved pricing mechanisms. In fact, there are always limitations on the performance of parametric modeling techniques and model misspecification is a major concern that can lead to erroneous valuations and hedging strategies.

In this paper, instead of attempting to improve option pricing models by introducing even more flexible models, we propose a method of improvement in an orthogonal direction. Our approach is based on the nonparametric correction of pricing errors induced by a given parametric model. We calibrate the chosen parametric model to best fit the observed option prices. Then we estimate and correct nonparametrically the pricing errors induced by the given model. Precisely, we estimate the state price survivor function, i.e., one minus the state price distribution, using a model-guided nonparametric procedure. We do so because European option prices can be easily expressed in terms of the state price survivor function. This function is always decreasing, has a water fall shape, and is in a neighborhood of a model-based distribution. We exploit this prior knowledge by estimating the main shape of the survivor function using a sensible parametric model. Then we correct the pricing errors induced by the model using a nonparametric approach. This method, called the Automatic

Correction of Errors (ACE) of a pricing formula, can be combined with any model-based pricing formula and reduces the pricing errors substantially. It does not depend sensitively on the initial model-based calibration, as the nonparametric method in the second step corrects the modeling biases in the initial step. In addition, compared to direct nonparametric methods, the guidance of model-based pricing formulae provides reliable initial options prices and exploits the rich developments in option pricing modeling, resulting in more accurate pricing methods.

Overall our method departs from the existing option pricing literature in two directions. Most existing studies focus on the state price density to price options. This density can be estimated nonparametrically by taking the second derivative of call prices but this procedure is numerically challenging. We rely on the state price distribution which is related to the first derivative of call prices and hence is easier to estimate. Then we fit nonparametrically pricing errors, or deviations from parametric option prices, instead of fitting option prices directly.

We use a nonparametric method to correct the pricing errors as functional forms of pricing errors are difficult to determine, varying over time and time to maturities. Nonparametric methods have the flexibility to discover the nonlinear relation between pricing errors and moneyness. In the nonparametric literature (e.g., Fan and Yao (2003)), it is well-known that survivor functions are easier to estimate, admitting a faster rate of convergence, than density functions. Hence we estimate the state price survivor function instead of the state price density. This is another important aspect of our methodological contribution to option pricing. The nonparametric learning and correction of pricing errors are easy to implement and fast to compute. The overall procedure is much faster than calibration based approaches such as Duan (1995), Bakshi, Cao, and Chen (1997), Heston and Nandi (2000), and Barone-Adesi, Engle, and Mancini (2008).

In the empirical analysis we consider European options on the S&P 500 index from January 2002 to December 2004. We compare our ACE method to (*i*) the benchmark ad hoc Black–Scholes model of Dumas, Fleming, and Whaley (1998), (*ii*) the parametric GARCH option pricing model of Heston and Nandi (2000), (*iii*) the semiparametric Black–Scholes model developed in this paper and inspired by Aït-Sahalia and Lo (1998), and (*iv*) the nonparametric regression approach applied directly to estimate the state price survivor function. We find that our ACE method outperforms both parametric models (ad hoc Black–Scholes and GARCH models) and semiparametric and nonparametric methods, in terms of fitting and prediction of option prices as well as hedging performance. Finally, we develop a nonparametric test based on the generalized likelihood ratio test of Fan, Zhang, and Zhang (2001) to document the efficacy of the ACE method. The test results show that the nonparametric correction of the ACE approach is very effective in reducing the pricing errors. These two pieces of empirical results provide stark evidence on the power of nonparametric learning of pricing errors for pricing options.

The paper is organized as follows. Section 2 introduces our ACE pricing method. Section 3 develops the nonparametric validation test. Section 4 recalls the semiparametric Black–Scholes and the GARCH pricing models. Section 5 presents the empirical results. Section 6 concludes.

2 Estimation of the state price survivor function

The basic idea in our approach is to estimate the state price survivor function using portfolios of traded options. Let S_t be the underlying asset price at time t and $f^*(\cdot)$ the state price (or risk neutral) conditional density of S_T given information at time t; e.g., Harrison and Kreps (1979). The dependence of the density $f^*(\cdot)$ on time t, time to maturity $\tau = T - t$, and other parameters are suppressed. Let C_t denote the price of the call option at time t, written on the asset S, with strike price X and time to maturity τ, whose payoff function is $\psi(S_T) = \max(S_T - X, 0)$. Then, C_t is the discounted expected payoff in the risk neutral world:

$$C_t = e^{-r_{t,\tau}\,\tau} E[\psi(S_T)] = e^{-r_{t,\tau}\,\tau} \int_X^\infty (y - X) f^*(y)\, dy,$$

where $r_{t,\tau}$ is the risk-free rate at time t for the maturity $T = t + \tau$. Let $F^*(x)$ be the cumulative state price distribution of S_T under the risk neutral measure, i.e., $F^*(x) = \int_0^x f^*(y)\, dy$. Integration by parts yields

$$C_t = e^{-r_{t,\tau}\,\tau} \int_X^\infty \bar{F}^*(y)\, dy, \tag{1}$$

where $\bar{F}^*(x) = 1 - F^*(x)$, is the state price survivor function of S_T.[1] Equation (1) has an interesting economic interpretation. Suppose n digital call options are available with strike prices $X + \delta, X + 2\delta, \cdots, X + n\delta$. Each of the digital options pays \$1 if the stock price at time T is larger than its corresponding strike price and zero otherwise. The forward price (before discounting) of each digital call option is $\bar{F}^*(X + i\delta)$, $i = 1, \cdots, n$. When δ is small and n is large, the portfolio Π of long positions δ in each of the digital call options has nearly the same payoff as the original call option, $\max(S_T - X, 0)$. Hence, in terms of forward prices,

$$E[\Pi] = \sum_{i=1}^{n} \bar{F}^*(X + i\delta)\,\delta \approx \int_X^\infty \bar{F}^*(y)\,dy = e^{r_{t,\tau}\tau} C_t.$$

This approximation suggests that the integral of the state price survivor function is indeed a portfolio of digital call options. In general, the price of any derivative contract with pay-off function $\psi(S_T)$ can be easily expressed in terms of the state price survivor function,

$$\int_0^\infty \psi(y) f^*(y)\,dy = \psi(0) + \int_0^\infty \bar{F}^*(y)\psi'(y)\,dy,$$

where ψ' is the first derivative of ψ. The pay-off function, ψ, needs to satisfy some mild regularity conditions that are usually verified by derivative contracts traded on the market.[2] Hence, for pricing purposes, knowing the state price survivor function is equivalent to know the state price density. Of course, if the final target is to price call options only, the call pricing function should be modeled directly. Our approach can be adapted also to this goal by using model-guided nonparametric techniques to estimate the call pricing function improving pricing accuracy. However, our method based on the state price survivor function is more general and can be used to price also other derivative contracts or less liquid options.

Let $F_{t,\tau} = S_t e^{(r_{t,\tau} - \delta_{t,\tau})\tau}$ be the forward price of the asset at time t, and $\delta_{t,\tau}$ the dividend yield paid by the asset between t and $T = t + \tau$. To avoid any confusions with state price distributions,

[1] Recently, Giacomini, Gottschling, Haefke, and White (2008) exploited this equality to price options with mixtures of t-distributions.

[2] The pay-off function, ψ, needs to be bounded at zero, i.e., $\psi(0) < \infty$; not increasing too rapidly, i.e., $\lim_{y \to +\infty} \psi(y)\bar{F}^*(y) = 0$; and the integral in the right hand side has to be well-defined.

we denote the forward price always using both subscripts, $F_{t,\tau}$. By the change of variable

$$C_t = e^{-r_{t,\tau}\,\tau} F_{t,\tau} \int_m^\infty \bar{F}(u)\,du, \tag{2}$$

where $m = X/F_{t,\tau}$ is called the moneyness and $\bar{F}(u) = 1 - F^*(F_{t,\tau}u)$ is the state price survivor function in the normalized scale, i.e., $F_{t,\tau}$ is normalized to \$1. For example the Black and Scholes (1973) model is characterized by the log-normal state price survivor function,

$$\bar{F}(m) = \int_m^\infty \frac{1}{u\sqrt{2\pi\sigma^2\tau}} \exp\left[-\frac{\left[\log(u)+\sigma^2\tau/2\right]^2}{2\sigma^2\tau}\right] du = 1 - \Phi\left[\frac{\log(m)+\sigma^2\tau/2}{\sigma\sqrt{\tau}}\right], \tag{3}$$

where σ is the constant volatility and Φ the standard Gaussian distribution function. Under this model, the option price in (2) can be explicitly calculated, resulting in the celebrated Black–Scholes pricing formula:

$$C_t = C_{\mathrm{BS}}(\sigma) = e^{-r_{t,\tau}\,\tau}\left(F_{t,\tau}\Phi(d_1) - X\Phi(d_2)\right), \tag{4}$$

where $d_1 = \left(\log(F_{t,\tau}/X) + \sigma^2\tau/2\right)/(\sigma\sqrt{\tau})$ and $d_2 = d_1 - \sigma\sqrt{\tau}$. In equation (4) we emphasize the dependence of the Black–Scholes option price on the volatility σ. In general, state price distributions have no closed-form expressions and numerical procedures are usually adopted in computations. For example, in jump diffusion models with nonparametric Lévy measure, Cont and Tankov (2004) suggest to approximate the state price density by using a mixture of log-normal distributions.

Using equation (2), pricing European options reduces to the estimation of the state price survivor function, $\bar{F}$. The parametric approach assumes a risk neutral dynamic for the underlying asset, and then derives the state price survivor function. For example, assuming that S_t follows a geometric Brownian motion implies the state price survivor function in equation (3). Then the parametric approach infers the model parameters from traded options. Our approach is nonparametric. We infer the state price survivor function directly from traded options. The key advantage of this method is that we need neither to assume a parametric model under the risk neutral measure, nor to derive an analytic form for the pricing formula. This eliminates model misspecification risk and allows fast estimation of the state price survivor function.

2.1 Traded options and state price survivor function

In this section, we discuss how to infer the state price survivor function from traded options. Let $X_1 < X_2$ be two consecutive strike prices of traded options with the same maturity. The portfolio of long positions in $(X_2 - X_1)^{-1}$ call options with strike X_1 and short positions in $(X_2 - X_1)^{-1}$ call options with strike X_2 has a pay-off function close to a digital call option with pay-off function $I(S_T > (X_1 + X_2)/2)$, where I takes values one if S_T exceeds $(X_1 + X_2)/2$, and zero otherwise; see Figure 1. Hence

$$e^{r_{t,\tau}\tau}\frac{C_t(X_1) - C_t(X_2)}{X_2 - X_1} \approx E[I(S_T > (X_1 + X_2)/2)] = \bar{F}^*((X_1 + X_2)/2),$$

where $C_t(X_i)$ is the price of the call option with strike X_i at time t. As derived in Appendix A, the mid-point approximation gives the best accuracy in terms of the order of the approximation error. We summarize the theoretical findings in the following proposition.

Proposition 1 *Let $C_t(X)$ be the price of the European call option with strike price X at time t. For two consecutive strike prices $X_1 < X_2$, we have*

$$e^{r_{t,\tau}\tau}\frac{C_t(X_1) - C_t(X_2)}{X_2 - X_1} = \bar{F}(\bar{m}_{t,1}) + O\left((m_{t,1} - m_{t,2})^3\right), \tag{5}$$

where $m_{t,i} = X_i/F_{t,\tau}$, $(i = 1, 2)$, is the moneyness and $\bar{m}_{t,1} = (m_{t,1} + m_{t,2})/2$. The approximation error is bounded by

$$-\frac{1}{24}\{\min_{m_{t,1}\leq\xi\leq m_{t,2}} f'(\xi)\}\,(m_{t,2} - m_{t,1})^3,$$

where $f'(\xi)$ is the derivative of the normalized state price density, i.e., $f'(x) = F''(x)$.

The proof is given in Appendix A. Proposition 1 provides a theoretical basis for inferring state price survivor functions from traded call options. State price distribution can be similarly recovered from the corresponding portfolio of traded put options. Equation (5) suggests that the state price survivor function can be recovered nonparametrically by taking the first derivative of the call price with respect to the strike price. To simplify the notation, let $C_{t,i}$ denote the call option price with moneyness $m_{t,i} = X_i/F_{t,\tau}$ at time t. Order the moneyness $\{m_{t,i}\}$ of traded call options at time t in an ascending order and denote by

$$\bar{m}_{t,i} = (m_{t,i} + m_{t,i+1})/2, \quad \text{and} \quad Y_{t,i} = e^{r_{t,\tau}\tau}\frac{C_{t,i} - C_{t,i+1}}{X_{i+1} - X_i}.$$

Then according to equation (5) we have

$$Y_{t,i} = \bar{F}(\bar{m}_{t,i}) + \varepsilon_{t,i}, \tag{6}$$

where $\varepsilon_{t,i}$ is the idiosyncratic noise. This approach reduces the pricing of options to a nonparametric estimation of the function $\bar{F}$, based on the observable data $\{(\bar{m}_{t,i}, Y_{t,i}), i = 1, \cdots, N_t\}$, where $N_t + 1$ is the number of options traded at time t for a given maturity. In contrast to other nonparametric methods such as those in Aït-Sahalia and Lo (1998) and Aït-Sahalia and Duarte (2003), our method estimates nonparametrically the state price survivor function rather than the state price density. Equation (5) shows that the former, $\bar{F}(m)$, is easily derived from options data. In contrast, the state price density is usually recovered by taking the second derivative of the call option function with respect to the strike price, which is a numerically challenging procedure; Breeden and Litzenberger (1978). Moreover, the state price distribution is much easier to estimate, admitting a faster rate of convergence than the state price density; e.g., Fan and Yao (2003).

The nonparametric method adopted here is the local linear regression. It has several advantages, including automatic boundary correction, high statistical efficiency, and easy bandwidth selection, as demonstrated in Fan (1992) and Fan and Gijbels (1995). For an overview of the local linear estimator and other related techniques, we refer the reader to Fan and Yao (2003). On a given day t_0, the nonparametric estimator of $\bar{F}$ is given by the time-weighted local linear regression

$$\min_{\beta_0,\beta_1\in\mathbb{R}^2} \sum_{t=t_0-d}^{t_0+d} \lambda^{|t_0-t|} \sum_{i=1}^{N_t} \left(Y_{t,i} - \beta_0 - \beta_1\left(\bar{m}_{t,i} - m\right)\right)^2 K_h\left(\bar{m}_{t,i} - m\right), \tag{7}$$

where $\lambda \in (0, 1]$ is the smoothing parameter in time, K is the kernel function, h is the bandwidth used to fit the local linear model, and $K_h(u) = h^{-1}K(u/h)$. Denoting by $\hat{\beta}_0$ and $\hat{\beta}_1$ the resulting minimizers, $\hat{\bar{F}}(m) = \hat{\beta}_0$ is the nonparametric estimate of the state price survivor function at moneyness m. With the estimated $\hat{\bar{F}}$, for example call option prices are computed via equation (2). The first summation in (7) aggregates options data on consecutive dates exploiting the time continuity of the state price survivor function. Without this time aggregation, the sample data available on t_0 only are likely not enough for an accurate estimation of $\bar{F}$. In our empirical application, we set $d = 2$, i.e., one trading week, achieving a sample size of 150–200 data points. Options traded on consecutive days have slightly different time to maturities and the time-weight, $\lambda^{|t_0-t|}$, accounts for this effect. The second summation in (7) is the standard local linear regression that approximates the function $\bar{F}(x)$ locally around a given point, m, by the linear function

$$\bar{F}(x) \approx \bar{F}(m) + \bar{F}'(m)(x - m) = \beta_0 + \beta_1(x - m)$$

for x in a neighborhood of m. The kernel weights $K_h(\bar{m}_{t,i} - m)$ are used to ensure that the regression is run locally. For instance, using the Epanechnikov kernel, $K(x) = \frac{3}{4}(1 - x^2)I(|x| \leq 1)$ that vanishes outside the interval $(-1, 1)$, the local linear regression in (7) uses only options data with moneyness $\bar{m}_{t,i}$ in the interval $m \pm h$. Hence the bandwidth controls the effective sample size. For example, Figure 2 shows the nonparametric estimate of the survivor function $\bar{F}$ on December 29, 2004, using $Y_{t,i}$s observed on December 27–31, 2004 with time maturities from 173 to 169 days. Visual inspection of the fitting would suggest that the nonparametric method performs well. Unfortunately, small errors in estimating the state price survivor function can translate into large pricing errors. Figure 3 shows that this is the case for the options traded on December 29, 2004.

2.2 Ad hoc Black–Scholes model

The implied volatility, $\sigma^{\mathrm{BS}}_{t,i} = C^{-1}_{\mathrm{BS}}(C_{t,i})$, is a common measure to represent the call price $C_{t,i}$. A well documented empirical feature of implied volatilities is the so-called volatility smile; e.g., Renault and Touzi (1996). For example, Figure 4 shows the implied volatilities of the call option prices analyzed in the previous section and observed on December 27–31, 2004. To account for this phenomenon and provide a benchmark option pricing model, Dumas, Fleming, and Whaley (1998) introduce an ad hoc Black–Scholes model where the implied volatilities are smoothed across moneyness by fitting a parabolic function:

$$\sigma^{\mathrm{BS}}_{t,i} = a_0 + a_1\, m_{t,i} + a_2\, m^2_{t,i} + \mathrm{error}_{t,i}, \tag{8}$$

where $\sigma_{t,i}^{\mathrm{BS}}$ denotes the implied volatility observed on day t for a given maturity T and moneyness $m_{t,i}$, $t \in [t_0 - d, t_0 + d]$, and $i = 1, \cdots, N_t + 1$. Implied volatilities observed on different days have slightly different time to maturities, $\tau = T - t$, for $t \in [t_0 - d, t_0 + d]$. In our empirical application, for each day t_0 and each maturity T, the quadratic function (8) is estimated using the least-squares regression with time-weight $\lambda^{|t_0 - t|}$, as in the nonparametric regression (7). To price an option with moneyness m and time to maturity τ, the fitted value, $\hat{\sigma}(m) = \hat{a}_0 + \hat{a}_1 m + \hat{a}_2 m^2$, is plugged in the Black–Scholes formula (4), obtaining an ad hoc Black–Scholes pricing formula $C^{\mathrm{BS}}(m) = C_{\mathrm{BS}}(\hat{\sigma}(m))$. Figure 4 shows the quadratic fit to the implied volatility. Some lack of fit is evidenced due to the inflexibility of the quadratic form. This translates into systematic pricing errors, as demonstrated in Figure 3.

Although theoretically inconsistent, ad hoc Black–Scholes methods are routinely used in the option pricing industry. They represent a challenging benchmark because they allow for different implied volatilities to price different options, and in our empirical applications they are separately fitted to each maturity. Moreover, Dumas, Fleming, and Whaley (1998) show that this model outperforms deterministic volatility function models introduced by Derman and Kani (1994), Dupire (1994), and Rubinstein (1994).

2.3 Model-guided nonparametric methods

The direct nonparametric approach (7) does not perform well because does not exploit the prior knowledge on the shape of the state price survivor function, uses a constant bandwidth h over the whole domain of $\bar{F}$, and does not account for the implied volatility smile explicitly. We propose to address these aspects by the following model-guided nonparametric estimation of the state price survivor function, $\bar{F}$.

To estimate the main shape of the survivor function, we combine the state price distribution (3) and the ad hoc Black–Scholes model (8). This results in the following preliminary estimator of the survivor function:

$$\bar{F}_{\mathrm{LN}}(m; \vartheta_t) = 1 - \Phi\left[\frac{\log(m) + (\hat{\sigma}(m)\,\vartheta_t)^2/2}{\hat{\sigma}(m)\,\vartheta_t}\right], \tag{9}$$

where $\hat{a}_0$, $\hat{a}_1$, $\hat{a}_2$ in $\hat{\sigma}(m)$ are estimated using the implied volatilities observed on $2d+1$ consecutive days as in equation (8). The parameter ϑ_t accounts for the slightly different time to maturities and is determined by minimizing the following distance:

$$\hat{\vartheta}_t = \arg\min_{\vartheta \in \mathbb{R}} \sum_{i=1}^{N_t} \left(Y_{t,i} - \bar{F}_{\mathrm{LN}}(\bar{m}_{t,i}; \vartheta) \right)^2 . \tag{10}$$

Notice that $\bar{F}_{\mathrm{LN}}(m; \vartheta_t)$ reduces to the Black–Scholes log-normal survivor function (3) when $a_1 = a_2 = 0$, and $\vartheta_t = \sqrt{T-t}$. The main shape of the survivor curve is now captured by our preliminary estimate $\bar{F}_{\mathrm{LN}}(m; \hat{\vartheta}_t)$. The accuracy of this estimate is not so important because it will be corrected by a nonparametric estimation in the second stage. Indeed, any state price survivor function, $\bar{F}_t(m)$, can be represented as

$$\bar{F}_t(m) = \bar{F}_{\mathrm{LN}}(m; \hat{\vartheta}_t) + \bar{F}_{t,c}(m), \tag{11}$$

where $\bar{F}_{\mathrm{LN}}(m; \hat{\vartheta}_t)$ is the parametric leading term of the state price survivor function and $\bar{F}_{t,c}(m)$ is the nonparametric correction term. Equations (6) and (11) give

$$Y_{t,i} = \bar{F}_{\mathrm{LN}}(\bar{m}_{t,i}; \hat{\vartheta}_t) + \bar{F}_{t,c}(\bar{m}_{t,i}) + \varepsilon_{t,i}. \tag{12}$$

This equation shows that $\bar{F}_{t,c}(m)$ is the regression function for the data $\tilde{Y}_{t,i} = Y_{t,i} - \bar{F}_{\mathrm{LN}}(\bar{m}_{t,i}; \hat{\vartheta}_t)$ on the moneyness $\bar{m}_{t,i}$. $\tilde{Y}_{t,i}$s are the pricing errors on the digital call options induced by the parametric model $\bar{F}_{\mathrm{LN}}(m; \hat{\vartheta}_t)$. These pricing errors are estimated and corrected by the nonparametric correction term $\bar{F}_{t,c}(m)$. This term can be estimated by the local linear fit to the data $\{(\bar{m}_{t,i}, \tilde{Y}_{t,i}), i = 1, \cdots, N_t; t \in [t_0 - d, t_0 + d]\}$ using the time-weighted nonparametric regression

$$\min_{\beta_0, \beta_1 \in \mathbb{R}^2} \sum_{t=t_0-d}^{t_0+d} \lambda^{|t_0 - t|} \sum_{i=1}^{N_t} \left(\tilde{Y}_{t,i} - \beta_0 - \beta_1 (\bar{m}_{t,i} - m) \right)^2 K_h (\bar{m}_{t,i} - m) . \tag{13}$$

Denoting by $\tilde{\beta}_0$ and $\tilde{\beta}_1$ the resulting minimizers, the nonparametric correction term is $\hat{\bar{F}}_{t,c}(m) = \tilde{\beta}_0$. The nonparametric regression is now applied to the $\tilde{Y}_{t,i}$s which are more homogeneous than the $Y_{t,i}$s, with approximately the same degree of smoothness as a function of m. Hence the constant bandwidth h should be a reasonable choice for estimating the correction term $\bar{F}_{t,c}(m)$. The shape of $\bar{F}_{t,c}(m)$ can vary over time and time to maturities and a nonparametric learning technique is particularly appealing. This procedure gives a model-guided nonparametric estimation of $\bar{F}_t(m)$ by plugging the nonparametric estimate of the function $\bar{F}_{t,c}(m)$ into (11). Nonparametric estimation of $\bar{F}_{t,c}(m)$ is equivalent to the nonparametric estimation of $\bar{F}_t(m)$. However, the former is easier to estimate nonparametrically than the latter because it is a smoother function of m. We remark that $\bar{F}_{t,c}$ is not a survivor function but the correction term of the parametric function $\bar{F}_{\mathrm{LN}}$. According to (2), the price of a call option can be summarized as follows.

Proposition 2 *The price of the European call option at time t with moneyness m and time to maturity τ can be decomposed as*

$$C_t = e^{-r_{t,\tau}\tau} F_{t,\tau} \int_m^\infty \bar{F}_{LN}(u; \hat{\vartheta}_t)\, du + e^{-r_{t,\tau}\tau} F_{t,\tau} \int_m^\infty \bar{F}_{t,c}(u)\, du. \tag{14}$$

The proof is simply given by substituting equation (11) into equation (2). Substituting $\hat{\bar{F}}_{t,c}(m)$ into (14), we obtain a new method for pricing derivatives instruments. The last term in (14) is the nonparametric correction of the pricing error induced by the parametric pricing formula. The overall procedure is still nonparametric and can be combined with any parametric approach. We refer to this method as the Automatic Correction of Errors (ACE) approach.[3]

As an example, Figure 2 shows the estimated state price survivor function on December 29, 2004 using the same call options as in previous sections and applying our ACE method (11) and the direct nonparametric method (7). Visually, both methods seem to provide a good fit to the data. However, Figure 3 shows that the pricing errors of the two methods behave very differently. The direct nonparametric approach does not perform well with a root mean square error (RMSE) of \$1.04. Our ACE approach reduces the pricing errors substantially and has a RMSE of only \$0.21. For completeness, Figure 3 also presents the pricing errors of the ad hoc Black–Scholes model (8). This method does not perform well with a RMSE of \$1.10, mainly because the fitted volatilities $\hat{\sigma}^{\mathrm{BS}}$ in equation (8) are not very accurate around the moneyness, $m \approx 1$. At-the-money options are very

[3] "Automatic" refers to the nonparametric fitting which does not need to impose any functional form. This type of parametric-guided approach has been used in the statistics literature; see, for example, Press and Tukey (1956) and Glad (1998).

sensitive to changes in volatilities and even small errors in the volatility estimation can induce large pricing errors. Figure 3 also shows the pricing performance of the semiparametric Black–Scholes model, to be introduced in Section 4.1, that is quite satisfactory with a RMSE of \$0.35.

2.4 Statistical properties of ACE and bandwidth selection

In this section, we show that the parametrically guided nonparametric fitting of the ACE method has smaller bias when the true state price survivor function is in a neighborhood of the parametric model $\bar{F}(m;\theta)$. The least-square calibration method gives the θ which minimizes

$$\sum_{t=t_0-d}^{t_0+d} \sum_{i=1}^{N_t} \left(Y_{t,i} - \bar{F}(\bar{m}_{t,i};\theta)\right)^2. \tag{15}$$

Let $n = \sum_{t=t_0-d}^{t_0+d} N_t$ be the sample size. If the true survivor function during the time period $[t_0 - d, t_0 + d]$ is $\bar{F}_0(m)$,[4] then the nonlinear least-square (15) attempts to find θ_0 which minimizes

$$E\left[\bar{F}_0(m) - \bar{F}(m;\theta)\right]^2. \tag{16}$$

The survivor function $\bar{F}(m;\theta_0)$ is the best approximation of the true state price survivor function, $\bar{F}_0(m)$, in the family of functions $\{\bar{F}(m;\theta)\}$. The following proposition summarizes the bias and variance of the estimator $\hat{\bar{F}}(m) = \bar{F}(m;\hat{\theta}) + \hat{\bar{F}}_c(m)$, where $\hat{\bar{F}}_c(m)$ is the local linear fit to the data $\{(\bar{m}_{t,i}, \tilde{Y}_{t,i})\}$.

Proposition 3 *Under the conditions given in Appendix B, we have*

$$\sqrt{nh}\{\hat{\bar{F}}(m) - \bar{F}_0(m) - \frac{1}{2}\bar{F}_c''(m)h^2 \int u^2 K(u)\,du - o(h^2)\} \xrightarrow{w} N(0, \sigma_\varepsilon^2(m) \int K^2(u)\,du/g(m)),$$

where $\bar{F}_c(m) = \bar{F}_0(m) - \bar{F}(m;\theta_0)$, $g(m)$ is the marginal density of the moneyness at the point m, and $\sigma_\varepsilon^2(m)$ is the conditional variance of $\varepsilon_{t,i}$ given $\bar{m}_{t,i} = m$.

The proof is given in Appendix B. The bias of the parametric-guided nonparametric estimator (i.e., the ACE method) has a leading term of order $\frac{1}{2}\bar{F}_c''(m)h^2 \int u^2 K(u)\,du$, while the direct nonparametric

[4]Here, for ease of presentation, we assume that the true survivor function is the same from $t_0 - d$ to $t_0 + d$, or varies very slowly in short time periods. If this assumption is violated, one needs to consider the date t_0 only, i.e., $d = 0$.

estimator has bias $\frac{1}{2}\bar{F}''(m)h^2\int u^2K(u)\,du$. The former is much smaller than the latter when $\bar{F}_c$ is smooth and small. This is particularly the case when $\bar{F}(m;\theta_0)$ is close to the true state price survivor function. The advantages of a parametrically guided nonparametric regression over the direct nonparametric approach are also documented in Glad (1998) and Fan and Ullah (1999). In our application, the curvature of $\bar{F}_0(m)$ is large when m is around one. Exploiting the shape of the function $\bar{F}(m;\theta_0)$, the curvature of $\bar{F}_c(m) = \bar{F}_0(m) - \bar{F}(m;\theta_0)$ can be significantly reduced. Hence the ACE method performs better than the direct nonparametric approach.

Hjort and Glad (1995) propose a similar method to ACE to estimate density functions. However, their goal is to estimate densities (not distributions) based on a random sample and they use a multiplicative decomposition of the unknown density function given by a parametric density times a nonparametric correction term, rather than an additive decomposition as in equation (11). Although our interest is to estimate the state price distribution, our problem is indeed reduced to a nonparametric regression problem inferred from option prices with different moneyness. We could also use a multiplicative decomposition of the state price survivor function in (11) but it would induce a less straightforward derivation of the adequacy test in Section 3. Garcia and Gençay (2000) introduce a somehow similar approach to price options deriving a generalized Black–Scholes formula but calibrated using neural networks.

Now we briefly discuss the issue of the bandwidth selection. Since the problem (11) is a standard nonparametric regression problem, a wealth of data-driven bandwidths can be employed; see Fan and Yao (2003). In particular, one can apply the pre-asymptotic substitution method of Fan and Gijbels (1995), or the plug-in method of Ruppert, Sheather, and Wand (1995). Alternatively, one can choose the bandwidth either subjectively or by a simple rule of thumb. The latter method takes nearly no computational cost and the selected bandwidth tends to be stable from one day to another, which is particularly important in practical applications. In our empirical study, we simply take $h = 0.3s$, where s is the sample standard deviation of the moneyness $\{\bar{m}_{t,i}, i = 1, \cdots, N_t; t \in [t_0 - d, t_0 + d]\}$. The standard deviation accounts for the spreadness of the moneyness and the constant factor 0.3 is an empirical choice from trial-and-error. Other constant factors might be required for other data sets.

The ACE method only requires univariate nonparametric estimations. Hence it can be easily applied using various bandwidths, selecting the bandwidth which induces the best pricing performance.

3 Adequacy of the pricing formula

Parametric pricing models are based on assumptions concerning the risk neutral dynamic of the underlying asset. Hence an important question is whether or not these assumptions are consistent with the observed option prices. In other words, does the pricing model induced by the parametric state price survivor function fit traded options adequately? Statistically, this is a nonparametric hypothesis testing problem:

$$H_0 : \bar{F}_t(m) = \bar{F}_t(m;\theta) \quad \longleftrightarrow \quad H_1 : \bar{F}_t(m) \neq \bar{F}_t(m;\theta), \tag{17}$$

based on the observed data from model (6), where $\bar{F}_t(m)$ is the true state price survivor function and $\bar{F}_t(m;\theta)$ is the state price survivor function derived from the parametric model. The null hypothesis is parametric but the alternative hypothesis is nonparametric. Hence the classical likelihood ratio test needs to be properly extended to deal with such a general situation. One of such extensions is the generalized likelihood ratio (GLR) test proposed by Fan, Zhang, and Zhang (2001). See Fan and Jiang (2007) for an overview. In the current setting, the GLR test compares the residual sum of squares when fitting model (6) using the parametric and the ACE methods. However, a direct application of the GLR statistic is not ideal here. Even when the null hypothesis is correct, the nonparametric fits incur biases; see Section 2.4. To improve the testing procedure, Fan and Yao (2003, Chapter 9) suggest to test whether or not the correction term $\bar{F}_{t,c}(m)$ is statistically away from zero. Under the null hypothesis, $\bar{F}_{t,c}$ is zero, or equivalently $\bar{F}_t(m) = \bar{F}_t(m;\theta)$, and the residual sum of squares is

$$\mathrm{RSS}_0 = \sum_{t=t_0-d}^{t_0+d} \sum_{i=1}^{N_t} \tilde{Y}_{t,i}^2 I(a \leq \bar{m}_{t,i} \leq b)$$

for a given large interval $[a, b]$. The survivor function is tested on the interval $[a, b]$ and the parameter θ characterizing $\bar{F}_t(m;\theta)$ is calibrated to traded options on the dates $[t_0 - d, t_0 + d]$ with moneyness falling in $[a, b]$. This procedure ensures that the test results are not driven by potential difficulties of the nonparametric approach in fitting the tails of the survivor functions. Under the alternative hypothesis, $\bar{F}_{t,c}$ is not zero and the residual sum of squares is

$$\mathrm{RSS}_1 = \sum_{t=t_0-d}^{t_0+d} \sum_{i=1}^{N_t} \left(\tilde{Y}_{t,i} - \hat{\bar{F}}_{t,c}(\bar{m}_{t,i})\right)^2 I(a \leq \bar{m}_{t,i} \leq b).$$

Let $n_{a,b} = \sum_{t=t_0-d}^{t_0+d} \sum_{i=1}^{N_t} I(a \leq \bar{m}_{t,i} \leq b)$ be the number of data points used in the fitting. The GLR test statistic is defined as

$$T_n = \frac{n_{a,b}}{2} \log(\mathrm{RSS}_0/\mathrm{RSS}_1) \tag{18}$$

and measures the inadequacy of the parametric fit. The larger the test statistics T_n, the less adequate the fit of the parametric model to options data. When T_n is very large (or beyond the usual high quantiles of its asymptotic null distribution), the null hypothesis that $\bar{F}_{t,c}$ is zero has to be rejected. The following proposition derives the asymptotic null distribution. We set $\lambda = 1$ for notational simplicity.

Proposition 4 *Under the conditions given in Appendix C,*[5] *if the null hypothesis is true, then*

$$r_K T_n \overset{a}{\sim} \chi^2_{a_n} \tag{19}$$

in the sense that

$$\frac{r_K T_n - a_n}{\sqrt{2a_n}} \xrightarrow{w} N(0,1),$$

where, with $*$ denoting the convolution operator,

$$r_K = \frac{K(0) - \int K^2(t)\,dt/2}{\int \left(K(t) - K*K(t)/2\right)^2 dt}, \quad a_n = s_K(b-a)/h + 1.45, \text{ and } s_K = \frac{\left(K(0) - \int K^2(t)\,dt/2\right)^2}{\int \left(K(t) - K*K(t)/2\right)^2 dt}.$$

The constants r_K and s_K are computed in Fan, Zhang, and Zhang (2001). For the Epanechnikov kernel, $r_K = 2.1153$ and $s_K = 0.9519$. The constant 1.45 in a_n comes from the empirical formula of Zhang (2003), who also demonstrates the adequacy of such an approximation.

In the above formulation, the parametric model $\bar{F}_t(m;\theta)$ can be any survivor function. In our empirical application, we take the log-normal survivor function (9) under the ad hoc Black–Scholes

[5]Similar assumptions are made by Aït-Sahalia and Lo (1998), and Gagliardini, Gourieroux, and Renault (2005), among others.

model (8). We apply the GLR test statistic (18) to the S&P 500 index options from January 2, 2002 to December 31, 2004, as detailed in Section 5. We set the coefficients a and b equal to the 0.05 and 0.95 quantiles of the observed moneyness for each relevant maturity. This procedure ensures that the estimate of the correction term is based on a sufficiently large sample size. Using (19) to compute the P-value, we find that all test statistics have a P-value no larger than 0.001, for every maturity in this three-year period. As a robustness check we computed P-values also using the conditional nonparametric bootstrap method in Fan and Yao (2003, Chapter 9) to better approximate the null distribution of the GLR test statistic. In nearly all tests P-values had similar and very low values. These results provide stark evidence that the nonparametric correction term, $\bar{F}_{t,c}$, is very effective in reducing the pricing errors of the ad hoc Black–Scholes model, $\bar{F}_{\mathrm{LN}}(m;\vartheta)$.

4 Other pricing methods

In the empirical study, we compare our ACE method also with the following two option pricing models.

4.1 Semiparametric Black–Scholes model

As demonstrated in Figure 4, the parabola in the ad hoc Black–Scholes model might not be flexible enough to fit the implied volatility smile. One way to overcome this difficulty is to fit the implied volatility function nonparametrically. This approach allows for a more flexible functional dependence of the implied volatility on moneyness, $\sigma^{\mathrm{BS}}(m)$. For each day t_0 and maturity T, we use the local linear regression to estimate the implied volatility function directly. As in the other model estimations, we aggregate options data around day t_0.[6] The local linear estimate, $\hat{\sigma}^{\mathrm{BS}}(m) = \hat{\beta}_0$, is given

[6] Aggregating the data around the date t_0 increases the sample size by an approximate factor of $2d+1$. Certainly, this reduces the variance of the resulting estimate. The time continuity of the pricing function implies that this aggregation does not introduce large estimation biases. Aggregation is really a time-domain smoothing resulting in a smoother estimated pricing function from one day to another. This is a desirable property in practical implementation.

by the time-weighted regression

$$\min_{\beta_0,\beta_1\in\mathbb{R}^2} \sum_{t=t_0-d}^{t_0+d} \lambda^{|t_0-t|} \sum_{i=1}^{N_t} \left(\sigma_{t,i}^{\mathrm{BS}} - \beta_0 - \beta_1\left(m_{t,i}-m\right)\right)^2 K_h\left(m_{t,i}-m\right) \tag{20}$$

and $\hat{\beta}_0$, $\hat{\beta}_1$ are the resulting minimizers. As in the ad hoc Black–Scholes model (8), call option prices are computed by plugging $\hat{\sigma}^{\mathrm{BS}}(m)$ in the Black–Scholes formula, $C_{t,i}^{\mathrm{BS}}$, and setting $\sigma = \hat{\sigma}^{\mathrm{BS}}(m)$. This pricing method is inspired by Aït-Sahalia and Lo (1998) who fit two-dimensional functions to the implied volatilities using a different nonparametric functional form. Figure 4 shows the nonparametric fit of the implied volatilities on December 29, 2004, using the same options as in the previous sections. The flexibility of the nonparametric fitting is evidenced.

4.2 GARCH option pricing model

Financial asset returns exhibit variances that change thorough time; e.g., Schwert (1989), and Jones (2003). The time varying volatility is often described using GARCH models; see Engle (1982), and Bollerslev (1986), and for surveys see, e.g., Bollerslev, Chou, and Kroner (1992) and Ghysels, Harvey, and Renault (1996). In our empirical analysis, we consider the parametric GARCH option pricing of Heston and Nandi (2000), as they derive an almost closed-form pricing formula. Here we only recall the main features of the model and we refer the reader to Heston and Nandi (2000) for a detailed description. Under the risk neutral distribution, the log-price follows the following GARCH model:

$$\begin{aligned} \log(S_t/S_{t-1}) &= r - h_t/2 + \sqrt{h_t}\, z_t \\ h_t &= \omega + \beta h_{t-1} + \alpha(z_{t-1} - \gamma\sqrt{h_{t-1}})^2, \end{aligned} \tag{21}$$

where z_t is a Gaussian innovation, h_t is the conditional variance of the log-return between $t-1$ and t, given the information, $\mathcal{I}_{t-1}$, available at time $t-1$. When $\beta + \alpha\gamma^2 < 1$, the log-return process is stationary with finite mean and variance. The parameters α and γ determine the kurtosis and the asymmetry of the distribution, respectively. When $\gamma > 0$, the model accounts for the so-called leverage effect, i.e., a negative shock z_t raises the variance more than a positive shock z_t of the same absolute magnitude. At time $t = 0$, the price of the call option with strike price X and maturity T is

$$\begin{aligned} C^{\mathrm{HN}} &= e^{-rT} E[\max(S_T - X, 0)] \\ &= e^{-rT}\zeta(1)\left(\frac{1}{2} + \frac{1}{\pi}\int_0^\infty \Re\left[\frac{X^{-i\phi}\zeta(i\phi+1)}{i\phi\,\zeta(1)}\right] d\phi\right) - e^{-rT}X\left(\frac{1}{2} + \frac{1}{\pi}\int_0^\infty \Re\left[\frac{X^{-i\phi}\zeta(i\phi)}{i\phi}\right] d\phi\right), \end{aligned} \tag{22}$$

where $\Re[\cdot]$ denotes the real part of a complex number, $i = \sqrt{-1}$, and $\zeta(\phi)$ is the moment generating function at time t of the log-price, $p_T = \log(S_T)$,

$$\zeta(\phi) = E[e^{\phi\, p_T} | \mathcal{I}_t] = e^{\phi\, p_t + A_t + B_t h_{t+1}}.$$

The coefficients A_ts and B_ts are computed backward using recursive equations; see Heston and Nandi (2000), and Christoffersen, Heston, and Jacobs (2006). Given the past underlying returns, the current variance, h_{t+1}, is known at time t. This is an important advantage of GARCH pricing models over other stochastic volatility models, such as the Heston (1993) and the Bakshi, Cao, and Chen (1997) models, where the instantaneous variance is not observed and requires a calibration or a separate estimation.

5 Empirical analysis

5.1 The data

We consider closing prices of European options on the S&P 500 index (symbol SPX) from January 2, 2002 to December 31, 2004. The data are downloaded from OptionMetrics. The average of bid and ask prices is taken as the option price.[7] To retain only liquid options (e.g., Aït-Sahalia and Lo (1998)), options with implied volatility larger than 70%, prices less than or equal to 1/8, or time to maturity (in calendar days) less than 20 days or more than 240 days are discarded, which yields a sample of 101,036 observations.

The market for SPX options is one of the most active index options market in the world. Expiration months are the three near-term months and three additional months from the March, June,

[7]Such mid prices are not transaction prices, which are not available in OptionMetrics, but they are all recorded at the same time simplifying somehow the analysis.

September, December, quarterly cycle. Strike price intervals are 5 and 25 points. The options are European, have no wild card features, and can be hedged using the active market on the S&P 500 index futures. Consequently, SPX options have been the focus of many empirical investigations, including Aït-Sahalia and Lo (1998), Chernov and Ghysels (2000), Heston and Nandi (2000), Carr, Geman, Madan, and Yor (2003), and Barone-Adesi, Engle, and Mancini (2008).

The term structure of default-free interest rates is also downloaded from OptionMetrics and the riskless interest rate for each maturity, τ_i, is obtained by linearly interpolating the two interest rates whose maturities straddle τ_i. This procedure is repeated for each contract and each day in the sample.

The raw data presents three challenges. First, in-the-money options are not actively traded compared to at-the-money and out-of-the-money options. For example, after the October '87 crash, the daily volume for out-of-the-money puts has been usually several times as large as the volume for in-the-money puts, reflecting the strong demand by portfolio managers for protective puts. Second, it is difficult to observe the underlying index price exactly when option prices are recorded. Temporal mismatches between option and index price recordings can induce pricing biases; e.g., Fleming, Ostdiek, and Whaley (1996). Third, the stocks in the S&P 500 index pay dividends and the future rates of dividend is difficult to determine. We address these three issues following the procedure suggested by Aït-Sahalia and Lo (1998). As option prices are recorded at the same time on each day, only one temporally matched index price per day is required to estimate a pricing model. As the dividend yield, $\delta_{t,\tau}$, is not observable, for each maturity τ the forward price, $F_{t,\tau}$, is computed via the put-call parity which holds because of the absence of arbitrage opportunities, independently of any option pricing model,

$$C_t + Xe^{-r_{t,\tau}\tau} = P_t + F_{t,\tau}e^{-r_{t,\tau}\tau},$$

where P_t denotes the put price. The forward price is computed using liquid calls and puts closest to at-the-money. The procedure is repeated for all dates and time to maturities. Then the prices of illiquid in-the-money calls are replaced by the corresponding prices implied by the put-call parity, $P_t + F_{t,\tau}e^{-r_{t,\tau}\tau} - Xe^{-r_{t,\tau}\tau}$, where the put price is out-of-the-money and therefore liquid. After this procedure the information in liquid out-of-the-money put prices translates into implied in-the-money call prices via the put-call parity. Hence put prices may be discarded without any loss of additional information. Some empirical studies investigate model pricing performances using calls and puts separately, and the corresponding empirical findings are rather similar; e.g., Bakshi, Cao, and Chen (1997), and Dumas, Fleming, and Whaley (1998).

We divide the call option data into several categories according to either moneyness or time to maturity. A call option is said to be deep in-the-money (DITM) if its moneyness $m < 0.8$; in-the-money (ITM) if $0.8 \leq m < 0.94$; at-the-money (ATM) if $0.94 \leq m < 1.04$; out-of-the-money (OTM) if $1.04 \leq m < 1.2$; and deep out-of-the-money (DOTM) if $m \geq 1.2$. An option contract can be classified, by the time to maturity, as short maturity ($<$ 60 days); medium maturity (60–160 days); and long maturity ($>$ 160 days). Table 1 describes the 101,036 call option prices and the implied volatilities used in the empirical analysis. The average call price ranges from \$354.06 for long maturity, DITM options to \$0.28 for short maturity, DOTM options. ITM, ATM, and OTM options account for, respectively, 20, 22, and 21 percent of the total sample. Short and long maturity options account for, respectively, 37 and 20 percent of the total sample. The table also shows the volatility smile and the corresponding term structure. For each set of maturities, the smile across moneyness is evident. The longer the time to maturity, the flatter the volatility smile.

5.2 Implementing the option pricing models

All nonparametric regressions use the local linear approximation and the Epanechnikov kernel. We set the bandwidth at $h = 0.3s$, where s is the sample standard deviation of the moneyness. We also experimented other similar bandwidth values and the overall pricing results for all nonparametric and semiparametric models were largely the same. In all time-weighted regressions, we set $\lambda = 0.83$ and measure the distance $|t - t_0|$ in calendar days. This choice assigns the weights 0.68, 0.83, 1, 0.83, 0.68 to options traded on the different week days, from Monday to Friday. We also experimented other similar values for λ and we obtained very similar results for all the pricing models.[8] As in the

[8]In practice the choice of λ is an empirical issue but the more options are available on day t_0 the lower λ should be.

GLR test (Section 3), the correction term $\bar{F}_{t,c}$ in the ACE method (11) is estimated from the 0.05 to the 0.95 quantiles of the observed moneyness and beyond this interval $\bar{F}_{t,c}$ is set to zero.

When implementing the GARCH pricing formula (22), the dividends paid by the stocks in the S&P 500 index have to be taken into account. For each maturity, we compute the dividend yield using spot-forward and put-call parities based on liquid closest at-the-money options, and then we subtract it from the current index level. All the other pricing methods are implemented using forward prices which already embed dividend yields.

5.3 In-sample model comparisons

Each Wednesday from January 2, 2002 to December 31, 2004, we calibrate the GARCH pricing model (21) to the cross section of options. Aggregating data over each week, each Wednesday and for each maturity we fit our ACE approach (11), the direct nonparametric model (7), the semiparametric Black–Scholes model (20), and the ad hoc Black–Scholes model (8). Then we price the European options available on each Wednesday obtaining 16,521 option price estimates for each model.

Table 2 summarizes the pricing errors of the five option pricing models across the different years from 2002 to 2004. Overall, the ACE method has the best pricing performance. For example, the ACE method has a RMSE 67% lower than the benchmark ad hoc Black–Scholes model. Tables 3 and 4 disaggregate dollar and relative pricing errors across the five moneyness and three maturities categories. The ACE method has the lowest RMSE in most comparisons. It has some difficulties in pricing deep in-the-money options and the reason is that to price options with low moneyness, such as $m \approx 0.4$, the survivor function has to be integrated for almost all the moneyness domain accumulating the estimation errors in $\bar{F}_t(m)$. Interestingly, the ACE method performs well in pricing at- and out-of-the-money options, which are more actively traded than in-the-money options. These results show that the two key characteristics of our method are very effective in producing accurate pricing results. On the one hand, they confirm that the space of state price distributions is a better space to price options than those of state price densities. It is so because using state price distributions avoids a numerical differentiation (performing instead an integration by parts) which induces a more stable numerical procedure. On the other hand, these results show that the model-guided nonparametric procedure allows to recover state price distributions precisely and hence pricing options accurately.

As shown in Tables 2, 3 and 4, the second best method is the semiparametric Black–Scholes model (20) (label Semip-BS), which performs particularly well for deep in-the-money options. Although the ad hoc Black–Scholes model (8) (label Ad Hoc BS) is estimated for each maturity, it does not perform well mainly because volatility smiles are not well approximated by parabolic functions. The semiparametric Black–Scholes model is designed to circumvent this problem and always outperforms the ad hoc Black–Scholes model. Table 10 shows summary statistics of the parabola coefficients and confirms that the volatility smile is a stable and persistent characteristic of implied volatilities.

The direct nonparametric model (7) (label NP) does not perform well, as anticipated, because does not exploit the prior knowledge on the shape of the state price survivor function, and a constant bandwidth is not adequate to estimate the survivor function over the whole domain. Comparing the ACE and NP methods show that the proposed bias reduction technique is very effective in reducing pricing errors. Tables 2, 3 and 4 provide stark evidence for the power of our idea of combining model-based pricing formulae and nonparametric learning to correct pricing errors.

The GARCH pricing model (21) (label GARCH) is not flexible enough to fit options with different maturities, when compared to the other methods. Table 11 shows the calibrated GARCH parameters and, as in previous studies (e.g., Heston and Nandi (2000), and Barone-Adesi, Engle, and Mancini (2008)), the parameter γ is largely positive confirming that negative shocks rise the volatility more than positive shocks of same absolute magnitudes. The GARCH parameters change over time, but the long run volatility and the persistency of the variance process, $\beta + \alpha\gamma^2$, are quite stable.

Figure 5 shows the absolute dollar and relative pricing errors across moneyness and maturities categories for the different pricing models.[9] The two proposed methods, the ACE and the semiparametric Black–Scholes methods, outperform all the other pricing models. In most occasions the ACE method outperforms the semiparametric Black–Scholes method.

Nonparametric estimations of state price survivor functions in the ACE method are guided by

[9]Given that the direct nonparametric model is largely outperformed by the ACE method, its graph is omitted.

the parametric model (9) which addresses the issue of volatility smile. An interesting question is how the ACE method behaves when a simpler and less accurate model is used as a parametric start. A naive parametric start is zero, resulting in the direct nonparametric method. This method does not work well, as demonstrated in Figure 3 and Tables 2–4. A more interesting question, raised by the referee, is the ACE with the Black–Scholes model as a parametric start. To answer this question, we repeated the previous in-sample (and the subsequent out-of-sample and hedging) analysis replacing model (9) by the log-normal survivor function as in the Black–Scholes model. To save space these additional results are omitted but are available from the authors upon request. As expected the overall performance of this ACE method is now less accurate as the Black–Scholes model is less accurate. Interestingly, this ACE method still outperforms the ad hoc Black–Scholes model and the direct nonparametric approach. This shows again that the nonparametric corrections are highly important and outperforms the parametric method that we initially use. This also indicates that a better parametric model puts less burden on the nonparametric correction and hence yields more accurate pricing. In particular, if the initial pricing formula is zero, the worst parametric model, the resulting method is direct nonparametric estimation. Hence, direct method is not expected to perform well. The empirical comparisons confirm the advantages of using model-guided nonparametric estimations over direct nonparametric estimations.

In the previous analysis the nonparametric and semiparametric methods are estimated on each Wednesday using also future information, namely options data on Thursday and Friday. This approach allows to exploit the time continuity of the pricing function, as for instance in Aït-Sahalia and Lo (1998). However, the previous methods can also be implemented using only current and past data. As a robustness check, we repeated the in-sample (and the subsequent out-of-sample and hedging) analysis pricing options on each Friday using data from Monday to Friday, without borrowing future information. These additional results (not reported here) largely confirm the reported findings and are available from the authors upon request. As a further robustness check, we repeated the in-sample (and the subsequent out-of-sample and hedging) analysis of the ACE method using an alternative kernel, namely the triweight kernel. The results are nearly the same as those based on the Epanechnikov kernel and are omitted but are available from the authors upon request.

5.4 Out-of-sample model comparisons

Out-of-sample pricing of options is an interesting challenge for any pricing method. It tests not only the goodness-of-fit of the pricing formula, but also whether or not the method overfits the option prices in the in-sample estimation. On each Wednesday and for each maturity, we use in-sample model estimates to price the same options one week later using forward prices, time to maturities and interest rates relevant on the next Wednesday.[10] In the ACE method, the model-based pricing formula, $\bar{F}_{\mathrm{LN}}(m;\vartheta_t)$, accounts for the shorter time to maturity. For this purpose, we fit the following regression model:

$$\hat{\vartheta}_t = \kappa\sqrt{T-t} + \mathrm{error}_t, \quad \text{for } t \in [t_0 - d, t_0 + d], \tag{23}$$

where $\hat{\vartheta}_t$s are given in (10). Equation (23) links time to maturities and $\hat{\vartheta}_t$s in the state price survivor function, $\bar{F}_{\mathrm{LN}}(m;\hat{\vartheta}_t)$. Using in-sample data observed over one trading week, the least-squares estimate gives

$$\hat{\kappa} = \frac{\sum_{i=1}^{5}\hat{\vartheta}_i\sqrt{\tau_i}}{\sum_{i=1}^{5}\tau_i}.$$

The coefficient $\hat{\vartheta}_{t_1} = \hat{\kappa}\sqrt{T-t_1}$ reflects the shorter time to maturity at the future date t_1. In our application $t_1 = t_0 + 7$ days.[11]

The out-of-sample pricing performances are summarized in Table 5 and disaggregated by moneyness and maturity in Tables 6 and 7; see also Figure 6. Interestingly, these results share the same pattern as in the in-sample analysis. The ACE method outperforms all the other pricing methods in terms of dollar and relative pricing errors, sometimes even by a larger extent. These results demonstrate that the automatic bias correction in the ACE is very effective. Without this part, the ACE method is essentially the same as the ad hoc Black–Scholes method, which does not perform well.

[10] The one week ahead forecast horizon is also adopted by Dumas, Fleming, and Whaley (1998) and Heston and Nandi (2000), among others.

[11] The coefficient $\hat{\vartheta}_{t_1}$ could be easily estimated by solving the minimization problem (10) using options data on time t_1 but this would invalidate the out-of-sample analysis.

These results also show that the second best method is our semiparametric Black–Scholes model, although the relative pricing errors increase when time to maturities decrease. Tables 5, 6 and 7 show that the GARCH model[12] has larger prediction errors than the ACE and the semiparametric Black–Scholes methods. These findings confirm the power of nonparametric approaches. The direct nonparametric model is omitted since it was largely outperformed by the ACE method in the in-sample analysis.

As a robustness check we repeated the previous out-of-sample analysis using one month horizon. The results share the same pattern as those reported in Tables 5, 6 and 7 and are omitted but are available from the authors upon request. In this out-of-sample analysis, the GARCH model becomes slightly more competitive, tends to outperform the ad hoc Black–Scholes model more often, but is still outperformed by our ACE method.

5.5 Hedging results

An important motivation for developing option pricing models is to provide better risk management of the derivatives assets. Setting hedge ratios based on accurate and reliable valuation model should induce an improvement in the hedging performance. Following Dumas, Fleming, and Whaley (1998), we evaluate the performance of a hedge portfolio formed on day t and liquidated one week later on day $t + 7$. The return on such a discretely adjusted hedge portfolio has three components: (*i*) the risk-free return on investment, (*ii*) the return from the discrete adjustment of the hedge, and (*iii*) the return from the difference between the change in the actual option price and the change in the theoretical option price over the week horizon; see Galai (1983), and Dumas, Fleming, and Whaley (1998). We use forward option prices, and hence the risk-free return component of the hedge portfolio is zero. As the focus is on model performance and not on the issues raised by discrete time rebalancing, we assume that the hedge portfolio is continuously rebalanced through time.[13] Hence

[12]In the out-of-sample pricing, the conditional variance, h_t, is updated using the risk neutral parameters calibrated at t_0 and the actual S&P 500 daily log-returns from t_0 to $t_0 + 7$ days.

[13]See, e.g., Bossaerts and Hillion (1997), and Bossaerts and Hillion (2003) for a hedging analysis in discrete time.

the hedging error is defined as

$$\epsilon_t = \Delta C_{\text{actual},t} - \Delta C_{\text{model},t}, \tag{24}$$

where $\Delta C_{\text{actual},t}$ is the observed change in the market option price from date t to date $t+7$, and $\Delta C_{\text{model},t}$ is the change in the model theoretical price. The proof of equation (24) is provided by Dumas, Fleming, and Whaley (1998, Section VI), but for completeness we recall it here as well. The hedging error resulting from the continuous time recalibration using the hedge ratio, ρ, is

$$\Delta C_{\text{actual},t} - \int_t^{t+7} \rho(S_u, u)\, dS_u. \tag{25}$$

If the valuation model gives the correct hedge ratio, ρ, to continuously rebalance the hedge, the two terms in (25) would be equal with probability one and the hedging error, ϵ_t, would be zero.

Table 8 summarizes the hedging errors of the four pricing models, and Table 9 disaggregates the hedging results across moneyness and maturity. These hedging results largely confirm the previous in- and out-of-sample pricing results. Our ACE approach tends to outperform all the other methods sometimes by a large extent. The overall second best method is our semiparametric Black–Scholes model. The ad hoc Black–Scholes model tends to dominate the GARCH model. Figure 7 shows the absolute hedging errors of the pricing models and visually confirms the previous results.

A further interesting exercise is to use the different models to hedge calendar spread portfolios. The rationale of this hedging analysis is that on each day t the ACE, semiparametric, and ad hoc Black–Scholes models are separately fitted to each time to maturity while the GARCH model is calibrated to the entire cross section of options, automatically imposing a time consistency across the estimated state price survivor functions. For each day t, we consider a calendar spread portfolio consisting of a long position in a call option with the longest time to maturity available in our database and a short position in a call option with same strike price and shortest available time to maturity. Then we assume that the portfolio can be continuously hedged over time and is liquidated one week later at time $t+7$. We compute hedging errors for all calendar spread portfolios available in our database and for each option pricing model. Overall these findings confirm the previous hedging results and are not reported but are available from the authors upon request. The hedging performance of the GARCH model seems to improve somehow when compared to previous hedging results but the flexibility of separately fitting options with different maturity outweighs the time consistency of the GARCH model.

6 Conclusions

We propose a new nonparametric method for estimating state price distributions and pricing financial derivatives. This method is called the Automatic Correction of Errors (ACE) in a pricing formula. The ACE approach is based on a model-guided nonparametric estimate of the state price survivor function. Given any parametric pricing model, the induced pricing errors are nonparametrically learned and corrected thorough the estimate of the survivor function, improving the model pricing performance. The ACE method is easy to implement and can be combined with any model-based pricing formula to correct the systematic biases of pricing errors. We also propose a semiparametric Black–Scholes method for option pricing, simplifying the method introduced by Aït-Sahalia and Lo (1998). Empirical studies based on S&P 500 index options show that the ACE approach outperforms, in terms of predictive and hedging abilities, the ad hoc Black–Scholes, the semiparametric Black–Scholes, the direct nonparametric, and the GARCH option pricing models. The proposed generalized likelihood ratio test show that the ACE method is very effective in reducing the pricing errors. Our ACE approach could be applied also in other contexts. For example, in credit risk modeling, accurate estimation of the default survivor function is essential in the pricing of credit risk sensitive contingent claims. This accuracy could be achieved by applying the ACE method.

A Proof of Proposition 1

Let $\bar{m} = (m_1 + m_2)/2$. By (2), omitting subscripts t, the left hand side of (5) can be expressed as

$$\frac{1}{m_2 - m_1} \int_{m_1}^{m_2} \bar{F}(u)\, du = \bar{F}(\bar{m}) + \frac{1}{m_2 - m_1} \int_{m_1}^{m_2} \big(\bar{F}(u) - \bar{F}(\bar{m})\big)\, du.$$

We now evaluate the approximation error. As $\bar{F} = 1 - F$, $\bar{F}' = -f$ and $\bar{F}'' = -f'$, by Taylor expansion to the second order, the second integral can be expressed as

$$\int_{m_1}^{m_2} \left(-f(\bar{m})(u - \bar{m}) - \frac{1}{2} f'(\xi)(u - \bar{m})^2 \right) du,$$

where ξ is a point lying between m_1 and m_2. The first term is zero, which is the advantage of using the mid-point, $\bar{m}$, and the second integral is bounded by

$$\frac{1}{2} \{ \max_{m_1 \le \xi \le m_2} -f'(\xi) \} \int_{m_1}^{m_2} (u - \bar{m})^2 du = -\frac{1}{24} \{ \min_{m_1 \le \xi \le m_2} f'(\xi) \} (m_2 - m_1)^3.$$

This completes the derivation of Proposition 1.

B Conditions and Proof of Proposition 3

To simplify the derivations, we make several idealizations. We assume that the data

$$\{(\bar{m}_{t,i}, Y_{t,i}), i = 1, \cdots, N_t; t \in [t_0 - d, t_0 + d]\}$$

are a sequence of i.i.d. random variables, satisfying model (6) with i.i.d. homoscedastic random noise, $\varepsilon_{t,i}$. In addition, we make the following technical assumptions.

(B1) The marginal density $g(\cdot)$ of the moneyness $\{m_{t,i}\}$ is continuous at the point m. The conditional variance $\sigma_\varepsilon^2(\cdot)$ of $\varepsilon_{t,i}$ is continuous at the point m.

(B2) $F_0(\cdot)$ has a continuous second derivative at the point m.

(B3) $E|\varepsilon|^{2+\delta} < \infty$ for some $\delta > 0$.

(B4) The function $K(t)$ is symmetric and has bounded support.

(B5) The bandwidth h tends to zero in such a way that $nh \to \infty$.

(B6) The function $F(x;\theta)$ is Lipschitz continuous in θ: $|F(x;\theta_1) - F(x;\theta_2)| \le C(x)|\theta_1 - \theta_2|$, with $C(x)$ bounded in a neighborhood of m. In addition, the calibrated $\hat{\theta}$ is root-n consistent.

From the definition of $\hat{\bar{F}}$ and $\bar{F}_c$, we have

$$\hat{\bar{F}}(m) - \bar{F}_0(m) = \bar{F}(m;\hat{\theta}) - \bar{F}(m;\theta_0) + \hat{\bar{F}}_c(m) - \bar{F}_c(m).$$

By condition (B6), the first difference term in the right hand side is of order $O_P(n^{-1/2})$. Hence,

$$\hat{\bar{F}}(m) - \bar{F}_0(m) = \hat{\bar{F}}_c(m) - \bar{F}_c(m) + O_P(n^{-1/2}). \tag{26}$$

We now deal with the main term in (26). Write the local linear regression smoother as

$$\hat{\bar{F}}_c(m) = \sum_{t=t_0-d}^{t_0+d} \sum_{i=1}^{N_t} W_{t,i}(m)\, \tilde{Y}_{t,i}, \tag{27}$$

where $W_{t,i}(m)$ is the weight induced by the local linear regression; see, for example, Fan and Yao (2003, §6.3.3). The local linear weights satisfy (Fan and Yao (2003, §6.3.3)),

$$\sum_{t=t_0-d}^{t_0+d} \sum_{i=1}^{N_t} W_{t,i}(m) = 1, \qquad \sum_{t=t_0-d}^{t_0+d} \sum_{i=1}^{N_t} |W_{t,i}(m)| \le 2 + o(1). \tag{28}$$

Then using (6) the data $\tilde{Y}_{t,i}$ can be written as

$$\tilde{Y}_{t,i} = \bar{F}_c(\bar{m}_{t,i}) + \varepsilon_{t,i} + Z_{t,i}, \tag{29}$$

where $Z_{t,i} = \bar{F}(\bar{m}_{t,i};\theta_0) - \bar{F}(\bar{m}_{t,i};\hat{\theta})$. By condition (B6), it follows that for a small neighborhood $\mathcal{N}$ around the point m,

$$\sup_{\bar{m}_{t,i}\in\mathcal{N}} |Z_{t,i}| = O_P(n^{-1/2}). \tag{30}$$

Since K has a bounded support, all data points that contribute to computing (27) fall in $\mathcal{N}$. Therefore, in (27) replacing $\tilde{Y}_{t,i}$ by (29) and using (28) and (30), we have

$$\hat{\bar{F}}_c(m) = \sum_{t=t_0-d}^{t_0+d} \sum_{i=1}^{N_t} W_{t,i}(m)\tilde{Z}_{t,i} + O_P(n^{-1/2}),$$

where $\tilde{Z}_{t,i} = \bar{F}_c(\bar{m}_{t,i}) + \varepsilon_{t,i}$. It follows from (26) that

$$\hat{\bar{F}}(m) - \bar{F}_0(m) = \hat{\bar{F}}_c^*(m) - \bar{F}_c(m) + O_P(n^{-1/2}),$$

where $\hat{\bar{F}}_c^*(m) = \sum_{t=t_0-d}^{t_0+d} \sum_{i=1}^{N_t} W_{t,i}(m)\tilde{Z}_{t,i}$ is the local linear regression smoother for the pseudo-data

$$\{(\bar{m}_{t,i}, \tilde{Z}_{t,i}), i = 1, \cdots, N_t; t \in [t_0 - d, t_0 + d]\}.$$

The result follows from the asymptotic normality theory of the local linear regression (Fan and Yao (2003)).

C Conditions and Proof of Proposition 4

We make the same idealized assumption as in Appendix B, that is

$$\{(\bar{m}_{t,i}, Y_{t,i}), i = 1, \cdots, N_t; t \in [t_0 - d, t_0 + d]\}$$

are a sequence of i.i.d. random variables, satisfying model (6) with i.i.d. homoscedastic random noise, $\varepsilon_{t,i}$. We assume further that

(A1) The marginal density of moneyness $\{m_{t,i}\}$ is bounded away from zero in the interval $[a, b]$.

(A2) $F(\cdot)$ has a continuous second derivative.

(A3) $E|\varepsilon|^4 < \infty$.

(A4) The function $K(t)$ is symmetric and bounded. Further the functions $t^3K(t)$ and $t^3K'(t)$ are bounded and $\int t^4K(t)\,dt < \infty$.

(A5) The bandwidth h satisfies $h \to 0$ and $nh^{3/2} \to \infty$.

(A6) The function $F(x; \theta)$ has a continuous derivative with respect to θ and the calibrated $\hat{\theta}$ is root-n consistent.

Under the above conditions, the number of observations in the interval $[a, b]$, $n_{a,b}$, by the Central Limit Theorem, is

$$n^{-1}n_{a,b} = P(m \in [a, b]) + O_P(n^{-1/2}),$$

where n is the total number of sample observations, i.e., $n = \sum_{t=t_0-d}^{t_0+d} N_t$. Let $T_n' = \frac{n}{2}\log(\mathrm{RSS}_0/\mathrm{RSS}_1)$, we have

$$T_n = n^{-1} n_{a,b}\, T_n' = P(m \in [a,b])\, T_n' + O_P(n^{-1/2} T_n'). \tag{31}$$

Now we can apply the result of Fan, Zhang, and Zhang (2001) to T_n', noting that the null hypothesis in our setting is $\bar{F}_{t,c} = 0$. In particular, according to the Remark 4.2 of Fan, Zhang, and Zhang (2001), we have $r_K' T_n' \overset{a}{\sim} \chi^2_{a_n'}$, where $r_K' = r_K P(m \in [a,b])$ and $a_n' = s_K(b-a)/h$. The last result and equation (31) imply that

$$r_K T_n = r_K' T_n' + O_P(n^{-1/2} h^{-1}) \overset{a}{\sim} \chi^2_{a_n}.$$

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		Maturity					
		Less than 60		60 to 160		More than 160	
		Mean	Std.	Mean	Std.	Mean	Std.
DITM	Call price $	323.22	89.16	342.00	108.12	354.06	117.30
	σ^{BS}%	43.91	9.70	35.67	7.82	30.57	5.40
	Observations	6,280		9,115		4,314	
ITM	Call price $	129.49	41.32	141.22	38.74	153.38	36.11
	σ^{BS}%	26.99	6.84	24.79	5.44	23.19	4.21
	Observations	7,969		8,609		3,840	
ATM	Call price $	30.61	18.60	45.47	19.45	64.27	19.20
	σ^{BS}%	18.06	5.82	19.16	5.01	19.36	4.08
	Observations	10,832		8,058		3,028	
OTM	Call price $	3.03	4.02	7.88	7.76	17.28	11.63
	σ^{BS}%	18.86	5.79	17.20	4.74	17.13	4.04
	Observations	7,395		9,175		4,302	
DOTM	Call price $	0.28	0.16	0.51	0.78	1.39	2.15
	σ^{BS}%	37.38	11.55	25.91	8.40	20.32	4.72
	Observations	4,561		8,775		4,783	

Table 1: Database description. The table shows mean, standard deviation (Std.) and number of observations for each moneyness and maturity category of SPX call option prices from January 2, 2002 to December 31, 2004, after applying filtering criteria and replacing illiquid in-the-money options as described in the main text. σ^{BS} is the Black–Scholes implied volatility. DITM is deep in-the-money options with moneyness less than 0.8, ITM is in-the-money options with moneyness between 0.8 and 0.94, ATM is at-the-money options with moneyness between 0.94 and 1.04, OTM is out-of-the-money options with moneyness between 1.04 and 1.2, and DOTM is deep out-of-the-money options with moneyness larger than 1.2. Moneyness is defined as strike price divided by the forward price of the underlying asset. Maturity is measured in calendar days.

Panel A: Aggregated valuation errors across all years

	Bias	RMSE	MADE	Min	Max	Err>0%	Bias%	RMSE%	MADE%
ACE	0.02	0.38	0.27	−1.81	2.02	51.62	0.92	8.20	3.60
Semip-BS	0.07	0.48	0.30	−3.44	3.05	57.91	1.92	9.62	4.01
Ad Hoc BS	0.17	1.14	0.80	−4.05	5.31	51.08	9.24	38.54	17.20
NP	0.62	1.19	0.82	−2.57	5.61	68.99	5.51	18.66	9.63
GARCH	−0.44	1.25	0.90	−7.99	6.27	27.14	−18.34	42.78	21.70

Panel B: Valuation errors by years

	Bias	RMSE	MADE	Min	Max	Err>0%	Bias%	RMSE%	MADE%
2002									
ACE	0.06	0.37	0.25	−1.81	2.02	54.20	1.02	9.80	4.78
Semip-BS	0.11	0.61	0.37	−3.44	3.05	58.22	2.13	11.42	5.55
Ad Hoc BS	0.10	1.15	0.77	−4.05	5.31	50.20	7.83	40.64	21.38
NP	0.83	1.35	0.89	−1.34	5.61	73.91	6.39	20.86	12.04
GARCH	−0.52	1.28	0.91	−7.99	4.36	22.36	−29.60	51.84	31.69
2003									
ACE	0.01	0.34	0.25	−1.69	1.43	51.44	0.91	8.27	3.64
Semip-BS	−0.02	0.36	0.24	−1.74	1.17	52.39	1.37	8.20	3.32
Ad Hoc BS	0.07	0.98	0.71	−3.30	4.08	48.96	7.22	30.91	14.55
NP	0.63	1.07	0.74	−1.04	4.27	72.18	4.98	16.02	8.60
GARCH	−0.39	0.93	0.72	−4.62	4.92	26.56	−20.21	42.06	21.36
2004									
ACE	0.01	0.42	0.31	−1.61	1.91	49.30	0.83	6.19	2.43
Semip-BS	0.12	0.43	0.28	−1.38	2.05	62.81	2.24	8.95	3.19
Ad Hoc BS	0.33	1.27	0.90	−3.15	5.02	53.92	12.50	42.73	15.70
NP	0.42	1.13	0.83	−2.57	3.93	61.26	5.18	18.75	8.29
GARCH	−0.40	1.45	1.06	−5.89	6.27	32.27	−5.78	32.65	12.45

Table 2: In-sample pricing errors. Bias, root mean square error (RMSE) and mean absolute error (MADE) of the dollar pricing error, (model price − market price), and of the percentage relative pricing error, $100 \times$ (model price − market price)/market price; Min (Max) is the minimum (maximum) dollar pricing error; Err>0% is the percentage of positive pricing errors, for the different pricing models and call option prices from January 2, 2002 to December 31, 2004.

		Maturity					
		Less than 60		60 to 160		More than 160	
		Bias	RMSE	Bias	RMSE	Bias	RMSE
DITM	ACE	−0.04	0.49	−0.03	0.47	0.06	0.55
	Semip-BS	−0.01	0.08	0.00	0.16	0.01	0.25
	Ad Hoc BS	−0.06	0.17	−0.18	0.43	−0.33	0.60
	NP	−0.24	0.59	−0.22	0.68	0.02	0.55
	GARCH	−0.39	0.84	−0.72	1.37	−1.08	2.63
ITM	ACE	−0.08	0.44	−0.13	0.42	−0.10	0.42
	Semip-BS	−0.00	0.33	0.04	0.48	0.06	0.63
	Ad Hoc BS	−0.54	0.78	−0.99	1.26	−0.94	1.26
	NP	0.15	0.72	0.32	0.91	0.59	1.01
	GARCH	−0.63	1.16	−1.41	1.66	−2.05	2.40
ATM	ACE	0.01	0.39	−0.01	0.38	0.01	0.37
	Semip-BS	0.11	0.62	0.16	0.74	0.16	0.82
	Ad Hoc BS	0.80	1.56	0.36	1.43	0.20	1.33
	NP	1.43	1.75	1.55	1.90	1.32	1.69
	GARCH	0.75	1.40	−0.11	0.73	−0.18	1.05
OTM	ACE	0.14	0.28	0.24	0.37	0.28	0.43
	Semip-BS	0.11	0.39	0.14	0.50	0.19	0.66
	Ad Hoc BS	0.89	1.29	1.17	1.51	0.92	1.44
	NP	0.73	1.11	1.07	1.40	1.21	1.54
	GARCH	−0.18	0.82	−0.60	0.89	0.01	0.93
DOTM	ACE	−0.02	0.06	0.01	0.09	0.06	0.14
	Semip-BS	0.00	0.07	0.02	0.11	0.01	0.20
	Ad Hoc BS	−0.06	0.19	0.12	0.37	0.16	0.35
	NP	−0.03	0.10	0.07	0.25	0.16	0.34
	GARCH	−0.29	0.32	−0.41	0.53	−0.52	0.65

Table 3: In-sample dollar pricing errors disaggregated by moneyness and maturity. Bias and root mean square error (RMSE) of the pricing error, (model price − market price), for the different models and call option prices from January 2, 2002 to December 31, 2004. See Table 1 for the definitions of DITM, ITM, ATM, OTM, and DOTM. Maturity is measured in calendar days.

		Maturity					
		Less than 60		60 to 160		More than 160	
		Bias	RMSE	Bias	RMSE	Bias	RMSE
DITM	ACE	−0.01	0.18	−0.01	0.16	0.01	0.18
	Semip-BS	−0.00	0.04	−0.00	0.07	0.00	0.10
	Ad Hoc BS	−0.03	0.08	−0.08	0.18	−0.13	0.24
	NP	−0.07	0.20	−0.06	0.23	0.02	0.20
	GARCH	−0.15	0.34	−0.28	0.51	−0.43	0.89
ITM	ACE	−0.08	0.39	−0.12	0.33	−0.08	0.29
	Semip-BS	−0.00	0.35	0.04	0.43	0.04	0.49
	Ad Hoc BS	−0.50	0.79	−0.77	1.04	−0.65	0.92
	NP	0.20	0.72	0.30	0.78	0.44	0.78
	GARCH	−0.49	1.02	−1.03	1.20	−1.33	1.55
ATM	ACE	0.78	2.69	0.21	1.25	0.09	0.76
	Semip-BS	1.02	4.12	0.56	2.25	0.32	1.41
	Ad Hoc BS	7.72	16.38	2.16	5.77	0.73	2.69
	NP	8.60	13.26	4.47	6.38	2.37	3.25
	GARCH	4.32	13.07	−0.00	2.18	−0.12	1.62
OTM	ACE	5.77	14.48	4.95	9.54	2.48	4.49
	Semip-BS	8.50	20.24	4.54	12.03	1.98	6.40
	Ad Hoc BS	54.33	88.42	34.56	57.92	10.95	21.00
	NP	26.02	43.59	18.18	26.86	8.96	12.51
	GARCH	−31.46	66.62	−24.90	38.96	−4.38	17.05
DOTM	ACE	−6.20	18.66	−0.74	16.20	1.84	11.62
	Semip-BS	1.70	17.77	4.54	16.72	2.39	12.84
	Ad Hoc BS	−25.84	60.48	6.38	57.80	13.27	40.12
	NP	−13.58	28.18	−0.02	24.55	4.84	17.39
	GARCH	−97.78	100.47	−91.64	93.28	−66.10	74.60

Table 4: In-sample percentage relative pricing errors disaggregated by moneyness and maturity. Bias and root mean square error (RMSE) of the relative pricing error, $100 \times$ (model price − market price)/market price, for the different models and call option prices from January 2, 2002 to December 31, 2004. See Table 1 for the definitions of DITM, ITM, ATM, OTM, and DOTM. Maturity is measured in calendar days.

Panel A: Aggregated valuation errors across all years

	Bias	RMSE	MADE	Min	Max	Err>0%	Bias%	RMSE%	MADE%
ACE	0.01	1.01	0.64	−5.31	9.81	51.29	0.87	24.96	11.44
Semip-BS	0.03	1.48	0.90	−7.21	6.89	44.66	−2.06	25.15	12.92
Ad Hoc BS	0.11	1.77	1.15	−7.86	7.17	45.62	6.69	47.66	22.05
GARCH	−0.49	2.27	1.44	−16.97	9.00	30.91	−18.69	48.20	25.66

Panel B: Valuation errors by years

	Bias	RMSE	MADE	Min	Max	Err>0%	Bias%	RMSE%	MADE%
2002									
ACE	−0.01	1.24	0.77	−5.31	9.36	48.93	0.17	31.64	16.95
Semip-BS	−0.22	1.93	1.21	−7.21	6.89	36.30	−7.90	32.68	19.97
Ad Hoc BS	−0.23	2.12	1.38	−7.86	6.97	38.73	−1.25	49.25	27.53
GARCH	−0.86	2.62	1.65	−14.58	8.63	22.46	−32.92	55.23	37.09
2003									
ACE	−0.06	0.88	0.54	−3.92	9.81	46.03	−1.69	21.63	9.64
Semip-BS	0.05	1.13	0.71	−5.55	4.14	47.52	−2.15	20.58	10.57
Ad Hoc BS	0.12	1.43	0.94	−6.21	5.09	46.42	4.77	36.87	18.00
GARCH	−0.39	1.74	1.16	−9.39	9.00	28.90	−21.01	43.69	23.57
2004									
ACE	0.10	0.90	0.61	−4.79	3.12	58.47	3.95	20.78	8.23
Semip-BS	0.24	1.30	0.81	−5.51	5.80	49.45	3.29	21.02	8.82
Ad Hoc BS	0.41	1.73	1.16	−4.99	7.17	51.04	15.66	54.84	21.00
GARCH	−0.27	2.38	1.53	−16.97	6.51	40.45	−3.66	45.45	17.39

Table 5: Out-of-sample pricing errors. Bias, root mean square error (RMSE) and mean absolute deviation error (MADE) of the dollar pricing error, (model price − market price), and of the percentage relative pricing error, $100 \times$ (model price − market price)/market price; Min (Max) is the minimum (maximum) dollar pricing error; Err>0% is the percentage of positive pricing errors, for the different models and call option prices from January 2, 2002 to December 31, 2004.

		Maturity					
		Less than 60		60 to 160		More than 160	
		Bias	RMSE	Bias	RMSE	Bias	RMSE
DITM	ACE	0.08	0.40	−0.08	0.52	−0.18	0.67
	Semip-BS	−0.08	0.25	−0.08	0.55	−0.02	0.88
	Ad Hoc BS	−0.11	0.30	−0.27	0.72	−0.37	1.02
	GARCH	−0.28	0.75	−0.85	2.04	−1.37	4.35
ITM	ACE	−0.18	0.71	−0.43	1.19	−0.40	1.40
	Semip-BS	−0.10	1.01	−0.08	1.58	−0.04	2.18
	Ad Hoc BS	−0.55	1.21	−1.10	1.94	−0.98	2.36
	GARCH	−0.55	1.22	−1.58	2.91	−2.39	5.40
ATM	ACE	0.05	1.17	0.01	1.58	−0.13	1.67
	Semip-BS	0.33	1.79	0.35	2.37	0.17	2.89
	Ad Hoc BS	0.87	2.26	0.44	2.59	0.20	3.08
	GARCH	0.76	1.87	−0.19	2.59	−0.51	4.58
OTM	ACE	0.27	0.74	0.44	1.30	0.48	1.25
	Semip-BS	−0.04	1.09	0.06	1.59	0.11	2.11
	Ad Hoc BS	0.58	1.49	1.04	2.09	0.83	2.60
	GARCH	−0.24	1.06	−0.76	1.74	−0.34	2.66
DOTM	ACE	−0.08	0.15	−0.01	0.27	0.11	0.38
	Semip-BS	−0.13	0.19	−0.09	0.32	−0.17	0.62
	Ad Hoc BS	−0.15	0.23	0.02	0.39	−0.03	0.66
	GARCH	−0.27	0.31	−0.43	0.62	−0.65	1.02

Table 6: Out-of-sample dollar pricing errors disaggregated by moneyness and maturity. Bias and root mean square error (RMSE) of the pricing error, (model price – market price), for the different models and call option prices from January 2, 2002 to December 31, 2004. See Table 1 for the definitions of DITM, ITM, ATM, OTM, and DOTM. Maturity is measured in calendar days.

		Maturity					
		Less than 60		60 to 160		More than 160	
		Bias	RMSE	Bias	RMSE	Bias	RMSE
DITM	ACE	0.02	0.16	−0.04	0.22	−0.07	0.27
	Semip-BS	−0.03	0.12	−0.03	0.25	−0.01	0.36
	Ad Hoc BS	−0.04	0.14	−0.12	0.32	−0.15	0.42
	GARCH	−0.11	0.31	−0.34	0.78	−0.56	1.58
ITM	ACE	−0.19	0.74	−0.34	1.07	−0.27	1.03
	Semip-BS	−0.09	1.05	−0.05	1.38	−0.03	1.63
	Ad Hoc BS	−0.50	1.25	−0.85	1.64	−0.67	1.75
	GARCH	−0.45	1.15	−1.15	2.25	−1.54	3.71
ATM	ACE	2.92	11.63	0.65	4.56	−0.03	2.82
	Semip-BS	4.01	14.64	1.54	6.77	0.53	4.91
	Ad Hoc BS	11.12	27.70	2.82	8.82	0.89	5.44
	GARCH	6.55	22.42	0.19	6.63	−0.39	7.17
OTM	ACE	8.53	45.11	11.32	27.59	5.48	11.31
	Semip-BS	−1.19	43.20	6.58	27.59	3.52	15.59
	Ad Hoc BS	46.17	99.17	36.30	71.46	12.35	28.19
	GARCH	−28.49	85.41	−24.21	41.23	−5.51	21.53
DOTM	ACE	−23.22	59.06	−6.63	48.31	4.87	24.52
	Semip-BS	−41.29	60.32	−14.06	42.23	−7.65	28.81
	Ad Hoc BS	−52.94	77.19	−8.61	67.63	2.91	49.15
	GARCH	−97.94	101.48	−92.26	93.98	−68.63	76.91

Table 7: Out-of-sample percentage relative pricing errors disaggregated by moneyness and maturity. Bias and root mean square error (RMSE) of the relative pricing error, $100 \times$ (model price − market price)/market price, for the different models and call option prices from January 2, 2002 to December 31, 2004. See Table 1 for the definitions of DITM, ITM, ATM, OTM, and DOTM. Maturity is measured in calendar days.

Panel A: Aggregated hedging errors across all years

	Mean	RMSE	MADE	Min	Max	Err>0%
ACE	−0.00	0.47	0.32	−3.01	2.73	51.61
Semip-BS	0.01	0.62	0.40	−4.07	3.00	53.35
Ad Hoc BS	−0.02	0.67	0.45	−4.11	3.15	51.92
GARCH	−0.06	1.15	0.73	−10.87	9.56	46.13

Panel B: Hedging errors by years

	Mean	RMSE	MADE	Min	Max	Err>0%
2002						
ACE	0.01	0.49	0.31	−3.01	2.47	52.60
Semip-BS	0.03	0.77	0.49	−4.07	3.00	56.14
Ad Hoc BS	−0.02	0.80	0.54	−4.11	2.75	55.82
GARCH	−0.07	1.22	0.76	−10.87	9.56	45.26
2003						
ACE	−0.02	0.41	0.29	−1.87	1.87	51.21
Semip-BS	−0.01	0.51	0.33	−2.13	2.03	50.40
Ad Hoc BS	−0.04	0.57	0.40	−2.63	2.01	49.46
GARCH	−0.06	0.89	0.56	−7.00	5.28	46.42
2004						
ACE	0.00	0.52	0.37	−1.92	2.73	51.02
Semip-BS	0.00	0.56	0.36	−2.42	2.94	53.43
Ad Hoc BS	−0.00	0.62	0.42	−2.19	3.15	50.42
GARCH	−0.05	1.28	0.86	−5.96	8.22	46.73

Table 8: Hedging error. Mean, root mean square error (RMSE) and mean absolute deviation error (MADE) of the dollar hedging error in equation (24); Min (Max) is the minimum dollar hedging error; Err>0% is the percentage of positive hedging errors, for the different models and call option prices from January 2, 2002 to December 31, 2004.

		Maturity					
		Less than 60		60 to 160		More than 160	
		Mean	RMSE	Mean	RMSE	Mean	RMSE
DITM	ACE	−0.01	0.73	0.01	0.65	0.04	0.71
	Semip-BS	0.00	0.13	0.01	0.23	0.03	0.37
	Ad Hoc BS	−0.00	0.19	0.00	0.31	0.02	0.44
	GARCH	−0.13	0.97	−0.02	1.30	−0.09	3.20
ITM	ACE	−0.02	0.64	−0.02	0.55	0.02	0.54
	Semip-BS	0.04	0.50	0.03	0.64	0.05	0.84
	Ad Hoc BS	−0.05	0.59	−0.02	0.69	0.08	0.86
	GARCH	−0.20	1.29	−0.21	0.91	−0.17	1.48
ATM	ACE	−0.01	0.48	−0.05	0.46	0.02	0.43
	Semip-BS	0.00	0.82	0.03	0.91	0.03	1.08
	Ad Hoc BS	−0.11	0.87	−0.03	0.87	0.05	1.02
	GARCH	0.03	1.33	−0.10	0.94	0.06	0.80
OTM	ACE	0.02	0.27	0.01	0.34	0.01	0.38
	Semip-BS	−0.05	0.52	−0.03	0.59	0.01	0.81
	Ad Hoc BS	0.01	0.69	−0.04	0.72	−0.03	0.85
	GARCH	0.02	0.88	0.02	0.72	0.00	1.02
DOTM	ACE	−0.01	0.08	−0.01	0.10	0.01	0.16
	Semip-BS	−0.01	0.13	−0.01	0.14	−0.00	0.24
	Ad Hoc BS	−0.01	0.24	−0.00	0.30	−0.01	0.29
	GARCH	−0.04	0.18	−0.04	0.23	−0.03	0.40

Table 9: Hedging errors disaggregated by moneyness and maturity. Mean and root mean square error (RMSE) of the dollar hedging error in equation (24) for the different models and call option prices from January 2, 2002 to December 31, 2004. See Table 1 for the definitions of DITM, ITM, ATM, OTM, and DOTM. Maturity is measured in calendar days.

	a_0		a_1		a_2	
Year	Mean	Std.	Mean	Std.	Mean	Std.
2002	0.84	0.09	−0.99	0.15	0.38	0.07
2003	0.76	0.08	−0.90	0.18	0.34	0.10
2004	0.81	0.10	−1.06	0.21	0.41	0.11

Table 10: Ad hoc Black–Scholes model (8), mean and standard deviation (Std.) of model parameters calibrated each Wednesday from January 2, 2002 to December 31, 2004, using call option prices.

	$\omega \times 10^{14}$		β		γ		$\alpha \times 10^6$		$\sqrt{E[h]}$		$\beta + \alpha\gamma^2$	
Year	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
2002	2.53	7.60	0.67	0.14	246.15	73.27	5.79	3.66	0.23	0.04	0.96	0.03
2003	1.07	3.96	0.66	0.22	284.69	93.57	5.48	6.89	0.22	0.03	0.95	0.07
2004	0.82	3.09	0.64	0.14	563.63	361.61	2.41	2.71	0.19	0.05	0.96	0.06

Table 11: Heston and Nandi GARCH model (21), mean and standard deviation (Std.) of model parameters (daily base) calibrated each Wednesday from January 2, 2002 to December 31, 2004, using call option prices. Risk neutral long run volatility (annual base) $\sqrt{E[h]} = \sqrt{365\,(\omega + \alpha)/(1 - \beta - \alpha\gamma^2)}$, and persistency of the variance process $\beta + \alpha\gamma^2$.

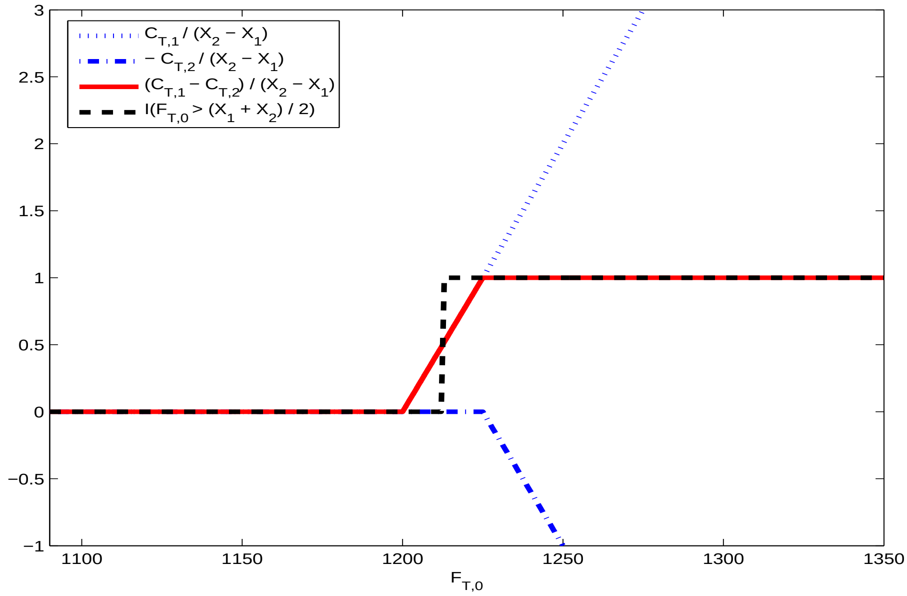

Figure 1: Pay-off function (solid line) of the portfolio: long 0.04 shares on the call option with strike price $X_1 = \$1{,}200$ and short 0.04 shares on the call option with strike price $X_2 = \$1{,}225$. This pay-off function can be approximated by an indicator function (dashed line). The dotted and dash-dotted lines are the pay-off functions of the long and short call options, respectively.

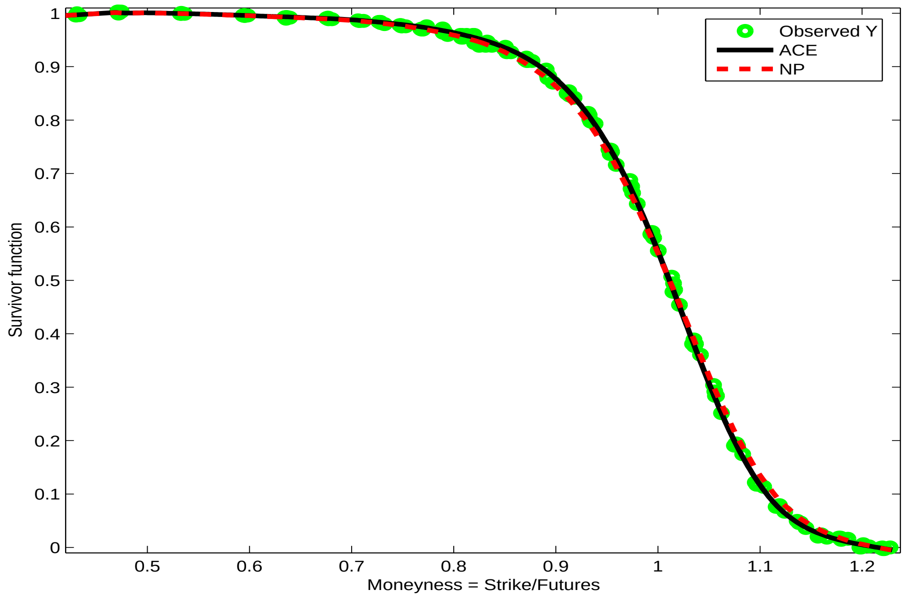

Figure 2: Scatter plot of $Y_{t,i} = e^{r_{t,\tau}\tau}(C_{t,i} - C_{t,i+1})/(X_{i+1} - X_i)$ versus the moneyness, $\bar{m}_{t,i}$, for the call option prices observed on December 27–31, 2004 with maturities 169–173 days. The survivor function is estimated by using the direct nonparametric approach (7) (NP) and the newly proposed ACE approach (11).

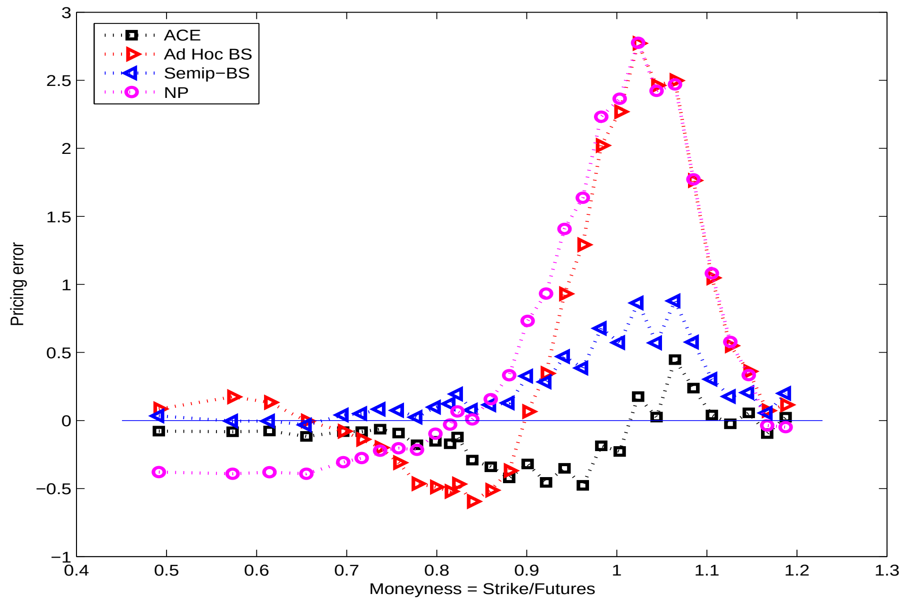

Figure 3: Pricing performance of the direct nonparametric approach (NP), the Ad Hoc Black–Scholes model (Ad Hoc BS), the ACE approach, the semiparametric Black–Scholes (semip-BS) model. The graph shows the dollar pricing error, (model price − market price), on December 29, 2004. All methods are estimated using call prices on December 27–31, 2004 with maturities 169–173 days.

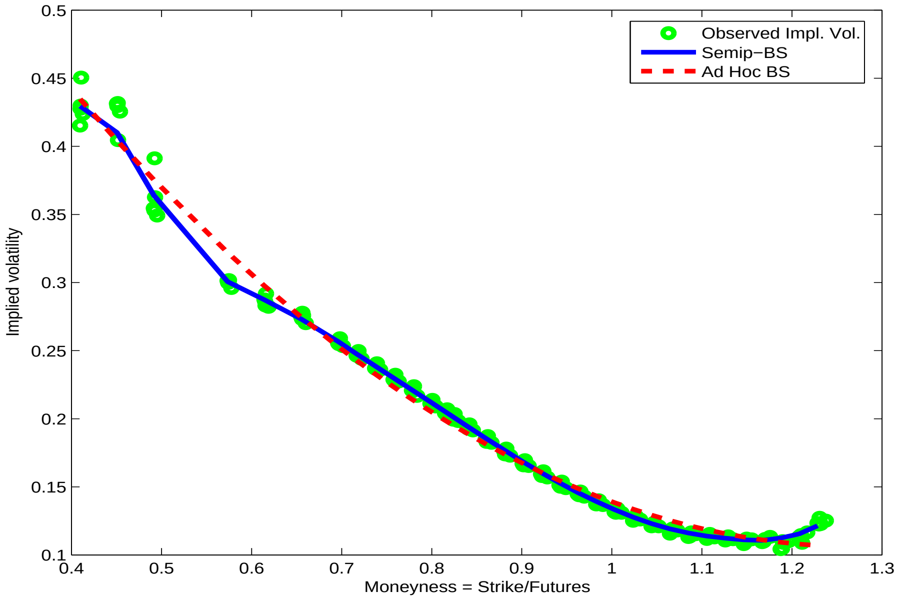

Figure 4: Implied volatilities observed on December 27–31, 2004 for call option prices with maturities 169–173 days. The figure shows the fitted parabola, $\sigma = a_0 + a_1 m + a_2 m^2$, (dashed line), and the local linear estimation of the function $\sigma(m)$, (solid line), where m denotes the moneyness.

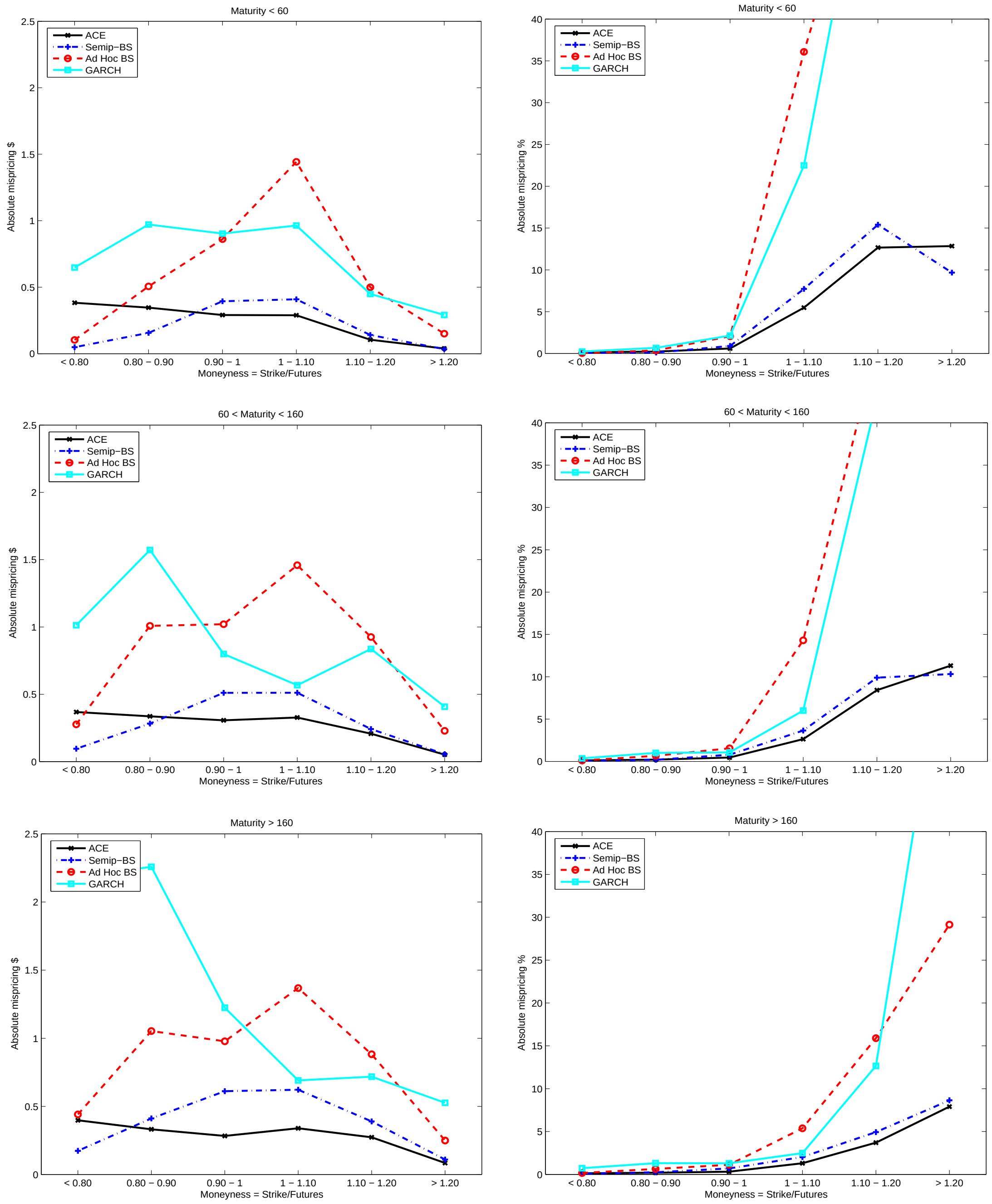

Figure 5: In-sample absolute mispricing in dollars, i.e., |model price − market price|, (left graphs), and in percentage, i.e., 100 × |model price − market price|/market price, (right graphs), for the different pricing models, averaged across the Wednesdays from January 2, 2002 to December 31, 2004 for the SPX call options.

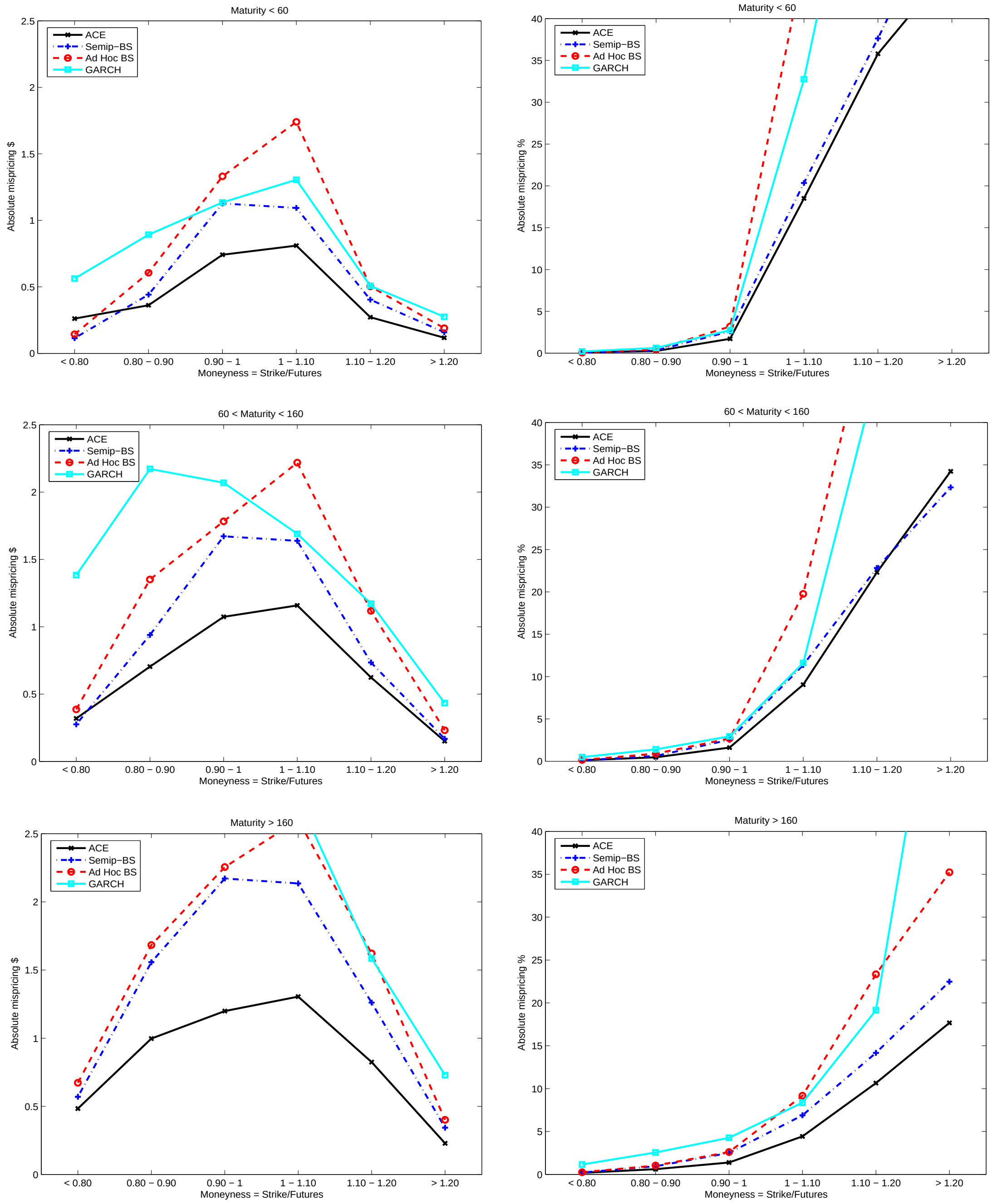

Figure 6: Out-of-sample absolute mispricing in dollars, i.e., $|\text{model price} - \text{market price}|$, (left graphs), and in percentage, i.e., $100 \times |\text{model price} - \text{market price}|/\text{market price}$, (right graphs), for the different pricing models, averaged across the Wednesdays from January 2, 2002 to December 31, 2004 for the SPX call options.

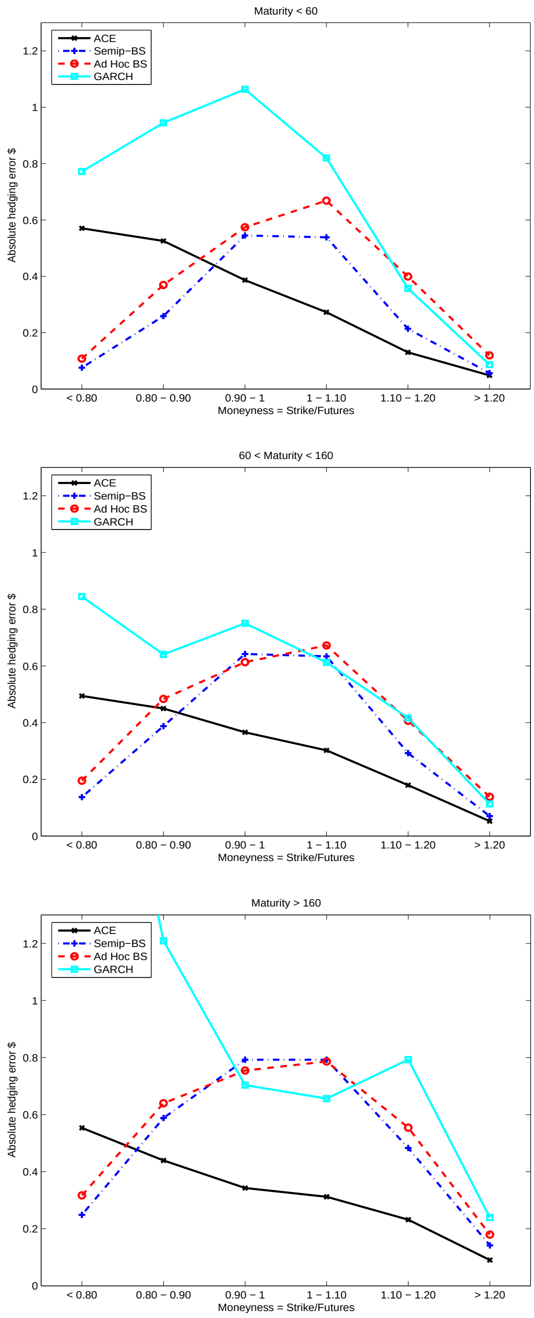

Figure 7: Absolute hedging error in dollars for the different pricing models, averaged across the Wednesdays from January 2, 2002 to December 31, 2004 for the SPX call options.