ORF/FIN 504: Financial Econometrics

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Problem Set #1 Spring 2025

Due Friday, February 7, 2025.

1. Download the daily, weekly and monthly prices for the Nasdaq index and the IBM stock from *Yahoo!Finance*. Reproduce Figures 1.3 – Figure 1.6 and Figure 1.8 in the lecture note using the Nasdaq index and the IBM stock data from January 2, 2011–December 31, 2018.

2. Suppose that the daily log-returns $\{r_t\}$of SP500 index have standard deviation 1%. For simplicity, log-returns will be referred to as returns.

   (a) What are the standard deviations (SDs) of montly and annual returns of the SP500 index?

   (b) Suppose that the expected annual log-return is 10%, what is the probability that the annual return is negative (you may assume that the annual returns are normally distributed)? What is the probability the the 4-year cumulative return of SP500 is negative during a US-president term? Empirically, how many U.S. Presidents have a negative returns of SP500 index during his tenure since 1980 (the last 11 President's terms)? **(You do not need to answer the last question)**

   (c) If $r_t \sim N(10/252, 1)$, how many years it is expected to take for us to see a negative 4% or more event in daily returns.

   (d) Now suppose that $r_t = 10/252 + \sqrt{1/2}t_4$ so that it has also 1% SD, how likely to get a negative 4% or more event under this t-distribution?

3. Generate a random sample of size 1000 from the t-distribution with degree of freedom ν and another random sample of size 1000 from the standard normal distribution. Apply the Kolmogorov-Smirnov test to see if they come from the same distribution. Report the value for $\nu = 5$, 10, 15 and 20.

4. Generate 1000 time series from the independent Gaussian white noise $\{r_t\}_{t=1}^T$ with $T = 100$. Compute

$$Z = \sqrt{T}\widehat{\rho}(1), \quad Q_m, \quad Q_m^*$$

   for $m = 3, 6$, and 12. Plot the histograms of Z, Q_3, Q_3^* and Q_6 and compare them with their asymptotic distributions. Report the 90^{th}, 95^{th}, and 99^{th} percentiles of the statistics $|Z|$, Q_3, Q_3^*, Q_6, Q_6^*, Q_{12} and Q_{12}^*, among 1000 simulations and compare them with their theoretical (asymptotic) percentiles. Repeat the experiment when $T = 400$.

5. Which of the following models are stationary. Find the roots of their characteristic polynomials and give the rates of decay for their acf functions when stationary.

   (a) AR(2): $X_t = 0.3X_{t-1} - 0.1X_{t-2} + \varepsilon_t$.

   (b) MA(2): $X_t = \varepsilon_t + 2\varepsilon_{t-1} - 5\varepsilon_{t-2}$.

   (c) ARMA(2,2): $X_t = -X_{t-1} + 6X_{t-2} + \varepsilon_t + 2\varepsilon_{t-1} - 5\varepsilon_{t-2}$.

6. Suppose that a stock return follows the nonlinear relationship $X_t = 0.8\varepsilon_{t-1}^2/(1 + \varepsilon_{t-1}^2) + \varepsilon_t$, and that $\{\varepsilon_t\} \sim_{i.i.d.} N(0, \sigma^2)$.

   (a) Simulate the time series of length 500 with $\sigma = 1$, and show the the plots of ACF and PACF.

   (b) Show that the ACF of $\{X_t\}$ is zero except at lag 0;

   (c) Use (b) to show that the PACF of $\{X_t\}$ is zero.

   This example shows that ACF and PACF are useful mainly for linear time series.

7. (**not required** to hand in; will not be graded) Consider a path dependent payoff function $Y_t = a_1 r_{t+1} + \cdots + a_k r_{t+k}$ where $\{a_i\}_{i=1}^k$ are given weights. If the return time series is weak stationary in the sense that $\mathrm{cov}(r_t, r_{t+j}) = \gamma(j)$. Show that

$$\mathrm{var}(Y_t) = \sum_{i=1}^{k}\sum_{j=1}^{k} a_i a_j \gamma(i-j).$$

   A natural estimate of this variance is the following substitution estimator:

$$\widehat{\mathrm{var}}(Y_t) = \sum_{i=1}^{k}\sum_{j=1}^{k} a_i a_j \widehat{\gamma}(i-j),$$

   where $\widehat{\gamma}(i-j)$ is defined by (1.8). Show that $\widehat{\mathrm{var}}(Y_t) \geq 0$.

8. (**not required** to hand in, *Implication of martingale hypothesis*; will not be graded). Let S_t be the price of an asset at time t. One version of the EMH assumes that the prices of any asset form a martingale process in the sense that

$$E(S_{t+1}|S_t, S_{t-1}, \cdots) = S_t, \quad \text{for all } t.$$

   To understand the implication of this assumption, we consider the following simple investment strategy. With initial capital C_0 dollars, at the time t we hold α_t dollars in cash and β_t shares of an asset at the price S_t. Hence the value of our investment at time t is $C_t = \alpha_t + \beta_t S_t$. Suppose that our investment is self-financing in the sense that

$$C_{t+1} = \alpha_t + \beta_t S_{t+1} = \alpha_{t+1} + \beta_{t+1} S_{t+1},$$

and our investment strategy $(\alpha_{t+1}, \beta_{t+1})$ is entirely determined by the asset prices upto the time t. Show that if $\{S_t\}$ is a martingale process, there exist no strategies such that $C_{t+1} > C_t$ with probability 1.